Choice Regularities

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- We introduce the notion of regularities to discipline the complexity of a choice axiom.
- We propose a novel approach (relative identification) to identify the simple behavioral differences that distinguishes a choice theory from another.

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- Theoretical results provide axiomatic characterizations of these models.
- However, used axioms are not as simple as WARP.
- This raises two questions:
  - **Q1.** Is it possible to obtain "simpler" characterizations?
  - **Q2.** Can we identify the behavioral differences among different choice procedures in a "simple" way?

Outline

Part 1: Regularities and some motivating observations.

Part 2: A relative identification exercise for a set of choice theories.

Revealed preference framework

A is an alternative set with n elements

choice sets are non-singleton $S \subset A$

choice space is the collection of all choice sets; Ω

a choice function is a mapping $c : \Omega \to A$ such that for each S, $c(S) \in S$

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Rational choice theory

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WARP: For each $S_1, S_2 \in \Omega$ such that $S_2 \subset S_1$ and $a \in S_2$,

$$\textit{if } a = c(S_1), \ \textit{then } a = c(S_2)$$

- a choice function c is rational iff c satisfies WARP (WARP identifies τ^{RC}).

first order regularity

Given c, a 1-reg(ularity) is a statement:

$$\text{if } a = c(S_1), \text{ then } b = c(S_2)$$

for some $S_1, S_2 \in \Omega$ and $a, b \in A$.

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Example:

if $a = c(a, b, c)$, then $a = c(a, b)$.

if $a = c(a, b)$, then $d = c(c, d)$.

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for some $S_1, S_2 \in \Omega$ and $a, b \in A$.

Definition: *A theory τ is 1-regular if one can find a collection of 1-regularities that identifies τ, i.e. a choice function $c \in \tau$ iff c satisfies all of these regularities.*

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- rational choice theory is 1-reg.

Let $A = \{a, b, c\}$

WARP $\equiv$

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- rational choice theory is 1-regular.

Observation: *If a proper choice theory that nests rational choice satisfies a 1-regularity* q*, then* q *must be in the* **form** *of WARP.*

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- Proof

Let $A = \{a, b, c\}$

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second order regularity

Given c, a 2-reg is a statement:

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for some $S_1, S_2, S_3 \in \Omega$ and $a, b, c \in A$.

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Definition: *A theory τ is 2-regular if one can find a collection of 2-regs that identifies τ, i.e. a choice function $c \in \tau$ iff c satisfies all of these regularities.*

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if $a = c(S_1)$ *and* $a = c(\{a, b\})$, *then* $b \neq c(S_2)$

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- wWARP identifies *Categorize then Choose* (MM, 2012) and *Rationalization* (CFS, 2012)
- So, we have 2-regular theories that nest rational choice.

simple=??? 2-regularity

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mth-order regularity

Given c, a m-reg is a statement:

$$a_1 = c(S_1) \wedge a_2 = c(S_2) \cdots \wedge a_k = c(S_m) \to a_{m+1} = c(S_{m+1})$$

for some $a_1, \ldots, a_{m+1} \in A$ and $S_1, \ldots S_{m+1} \in \Omega$.

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Definition: *A theory τ is m-regular if one can find a collection of m-regs that identifies τ, i.e. a choice function $c \in \tau$ iff c satisfies all of these regs.*

Predicate calculus

an m-reg is a formula:

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Our language:

the alphabet: a, S

the predicate (atomic statement): $a = c(S)$

logical connectives: $\wedge, \rightarrow$

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Any arbitrary choice theory can be identified if we verify high enough degree of regularities.

Observation: *Any choice theory is* $(2^n - n - 2)$*-regular where* n *is the number of alternatives.*

Proof

Relative Identification by 2-regularities

Relative identification???

relative identification

Definition: *Let τ_1 and τ_2 be two choice theories, a set of 2-regularities Q identifies τ_1 relative to τ_2 if*

1. *each $c_1 \in \tau_1$ satisfies each $q \in Q$,*
2. *for each $c_2 \in \tau_2 \setminus \tau_1$ there exists $q \in Q$ that c_2 fails to satisfy.*

relative identification

Definition: *Let* $\mathscr{F} = \{\tau_1, \tau_2, \ldots, \tau_k\}$ *be a family of choice theories. A set of 2-regularities Q relatively identifies* $\mathscr{F}$ *if for each* $\tau_i, \tau_j \in \mathscr{F}$*, Q identifies* τ_i *relative to* τ_j*.*

A family of choice theories

- Sequential rationality with binary rationales (Manzini & Mariotti'07, Apestegua & Ballester'09)
- Choice with Limited Attention (MNO'13)
- Choice by Game Trees (Xu & Zhou'07)
- List Rational Choice (Yildiz'12)

Question: Can we identify the behavioral differences among different choice procedures in a "simple" way?

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Answer: This may be achieved if we can seperate each choice theory from others via 2-regularities.

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Primitives: A collection of binary comparisions: $\succeq_1, \succeq_2, \ldots \succeq_k$

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Example:

$$a \succ_1 b;\ b \succ_2 c;\ a \succ_3 c$$

$$c(\{a, b, c\}) = a$$

Choice with limited attention

Primitives: an attention filter Γ and a preference $\succeq$.

$$c(S) = \quad \Gamma(S)$$

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List rational choice

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- a binary relation P used to compare pairs of alternatives

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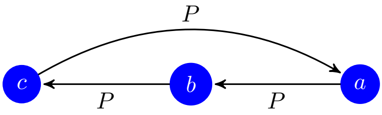
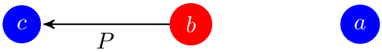
Example:

Suppose $a\ P\ b\ P\ c\ P\ a$




c
b
a


c
b
a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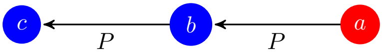
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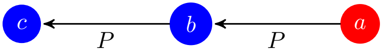
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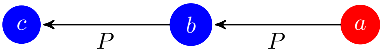

If x is the last alternative in S according to the list, then $c(S) = c(c(S \setminus x), x)$

Choice by game trees

Primitive: An extensive form game G.

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Player 1: $a \succ_1 b \succ_1 c$

Player 2: $b \succ_2 c \succ_2 a$

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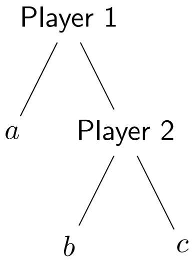

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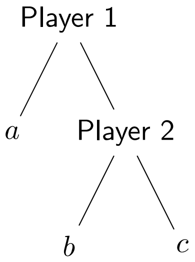

$$c(S) = SPNE(G|S) = a$$

A family of choice theories

- Sequential rationality with binary rationales: $c(S) = max(max(S, \succ_1), \succ_2)$
- Choice with Limited Attention: $c(S) = max(\Gamma(S), \succeq)$
- Choice by Game Trees: $c(S) = SPNE(G|S)$
- List Rational Choice: if x is the last alternative in S according to the list, then $c(S) = c(c(S \setminus x), x)$

Q1 What are the 2-regs satisfied by each theory?

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Q2 Would these 2-regs be sufficient to identify each theory in this family relative to the others?

2-regular axioms

Weak Path Independence (WPI): For each $S \in \Omega$, and $a, b \in S$,

if $a = c(S)$ and $b = c(S \setminus \{a\},)$ then $a = c(a, b)$

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No Dominated Choice (NDC): For each $a, b, c \in A$, and $S \in \Omega$ such that $a, b \in S$,

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Rival Monotonicity (RM): For each distinct $a, b \in A$ and $S_1, S_2 \in \Omega$ such that $S_2 \subset S_1$ and $b \in S_2$,

$$\text{if } a = c(S_2) \text{ and } b = c(S_1), \text{ then } a = c(S_2 \setminus \{b\})$$

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Binary Expansion (BE): For each $a, b, c \in A$,

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Path Existence (PE): For each $a, b, c \in A$,

$$\text{If } a = c(a,b,c) \text{ and } b = c(a,b), \text{ then } c = c(b,c)$$

Proposition: *Let $\mathcal{F}$ consist of the following boundedly rational choice theories: Rationalization via Game Trees, Sequentially Rational Choice , Revealed Attention, and List Rational Choice. Let Q consist of all the second order regularities in the form of WPI, NPC, RM, BE or PE.*

i. *For each $\tau \in \mathcal{F}$, if q is a second order regularity that τ satisfies, then $q \in Q$.*

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i. *For each $\tau \in \mathcal{F}$, if q is a second order regularity that τ satisfies, then $q \in Q$.*

ii. *Following table shows the second order regularities that each theory satisfies, and it follows that Q relatively identifies $\mathcal{F}$.*

	WPI	NDC	BE	RM
GT	$+$	$+$	$+$	$-$
SRC	$-$	$+$	$+$	$-$
RA	$-$	$-$	$-$	$-$
LRC	$+$	$+$	$+$	$+$

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Corollary: *Each choice theory in our family is identified relative to the others by 2-regularities.*

GT vs. SRC

	WPI	NDC	BE	RM
GT	$+$	$+$	$+$	$-$
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Weak Path Independence: For each $S \in \Omega$, and $a, b \in S$,

if $a = c(S)$ and $b = c(S \setminus \{a\})$ then $a = c(a, b)$.

GT vs. LRC

	WPI	NDC	BE	RM
GT	$+$	$+$	$+$	$-$
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	WPI	NDC	BE	RM
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Rival Monotonicity : For each distinct $a, b \in A$ and $S, T \in \Omega$ such that $S \subset T$ and $b \in S$,

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- These differences are difficult to infer from the existing axiomatizations.
- Any other behavioral difference can only be represented as higher regularities.

Further Observations

Gain from axiomatizations

- Any choice theory is $(2^n - n - 2)$-regular.
- It follows from the axiomatizations that the *theory of revealed attention* and *list rational choice theory* are $(2n - 1)$-regular.

Further Observations

Let τ_1 be a r_1 regular choice theory, and τ_2 be a r_2 regular choice theory.

- The choice theory $\tau_1 \cup \tau_2$ is $r_1 + r_2 + 1$ regular.
- The choice theory $\tau_1 \cap \tau_2$ is $max\{r_1, r_2\}$ regular.

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THANK
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Step 2: Now, we must have $a = b$, otherwise any rational choice function that chooses a from S_1 would not satisfy q. □

Back

List Rational Choice

For each distinct $x, y \in A$, x is revealed to follow y ($x \; F_c \; y$), if for some $S \in \Omega$, we have either

(i) $x = c(S \cup y)$ and $[y = c(x, y)$ or $x \neq c(S)]$ or

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Suppose there exist $c \in \tau$ and $c' \notin \tau$ such that c satisfies the precedent part of the the statement $q_{c'}$, but not the consequent part. By the construction of $q_{c'}$, this means for each $S \in \Omega$, $c(S) = c'(S)$. Hence we obtain a contradiction. $\square$

A better anology

Expected Utility

For each $f, g \in \mathcal{F}$,

$f \succeq g$ iff $E_\mu(f) \geq E_\mu(g)$

Maxmin Expected Utility:

For each $f, g \in \mathcal{F}$,

$f \succeq g$ iff $min_{\mu \in \mathscr{M}} E_\mu(f) \geq min_{\mu \in \mathscr{M}} E_\mu(g)$

Let $\succeq$ be a non-trivial, monotonic, Archmidean weak order;

Expected Utility: $E_\mu(f)$

iff

Independence: $\forall f, g, h \in \mathcal{F}$, $\forall \lambda \in (0,1)$,

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C-Indep.: $\forall f, g \in \mathcal{F}$ and certain act h, $\forall \lambda \in (0,1)$,

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Uncertainty Aversion: $\forall f, g \in \mathcal{F}$,

$$f \sim g \Rightarrow \frac{1}{2}f + \frac{1}{2}g \succeq f, g$$