Implementation via Codes of Rights

Semih Koray * Kemal Yildiz **

*Bilkent University

**New York University

example

Pareto Rule with Minimal Liberalism (PwL):

- a society, $N = \{1, 2, 3\}$, with individual preferences on a, b, c
- for each alternative $x \neq a$, x is (socially) acceptable iff x is Pareto efficient
- a is acceptable iff a is not only Pareto efficient, but also agent 1 prefers a to b

implementation problem

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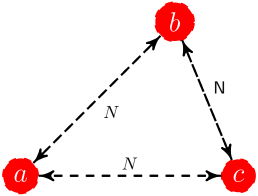
to implement PwL , for each $x, y \in \{a, b, c\}$

- entitle N to move from x to y
- additionally, entitle 1 to move from a to b



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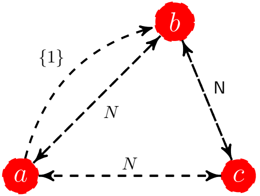

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observation: the resulting general game forms are quite different from (more complicated than) the real life mechanisms that we observe we aim to formulate and analyze a framework for implementation written in a language closer to the institutional real life mechanisms

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Nash implementation is the benchmark model for classical implementation

- used mechanisms are complicated
- characterization conditions are not easy to verify
- asymmetric conditions apply for two agents
- many interesting rules are not Nash-implementable

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- Subgame Perfect Nash implementation, **Moore** & **Repullo (1988)**
- Undominated Nash implementation, **Palfrey** & **Srivastava (1991)**
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- Implementation with Partially Honest Individuals, **Dutta** & **Sen (2012)**

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a persistent criticism: used mechanisms are difficult to interpret

preliminaries

A is a non empty finite set of alternatives

N is the set of agents with n elements

each non empty $K \subset N$ is a coalition

u_i denotes the preference relation of agent i
u is a preference profile of N on A

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a social choice rule, F, maps each u to a set of acceptable alternatives $F(u)$

Nash implementation

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an SCR F is Nash implementable via a game form (M, g) if

$$\text{for each } u \in \mathcal{P},\ F(u) = \{g(m) : m \in NE(M, g, u)\}$$

rights structure, Sertel (2001)

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- S is the set of (social) states
- an outcome function $h : S \to A$
- a code of rights γ maps each (distinct) state pair (s, t) to a coalition family

a coalition $K \in s \xrightarrow{\gamma} t$ is interpreted as K is given the right to move from s to t

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i) a state can be an alternative $(S = A)$, then

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ii) a state can be a proposal for an alternative with some supporting evidence, then

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a coalition $K \in s \xrightarrow{\gamma} t$ is interpreted as K can refute the evidence for s

Given a pair (S, h) and a preference profile u, for each distinct (s, t), a coalition K prefers to move from s to t, denoted by $K \in s \xrightarrow{u} t$, iff

$$\text{for each } i \in K, h(t)\ u_i\ h(s)$$

Γ-implementation

Definition: For each $u \in \mathcal{P}$, a state s is an equilibrium of rights structure Γ at u if for each $t \in S$,

$$(s \xrightarrow{\gamma} t) \ \cap \ (s \xrightarrow{u} t) = \emptyset$$

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Definition: F is Γ- implementable if there is a rights structure $\Gamma = (S, h, \gamma)$ such that,

$$\text{for each } u \in \mathcal{P}, \ F(u) = h(E(\Gamma, u))$$

majority rule

- let $N = \{1, 2, 3\}$ and $A = \{I, G\}$
- $F(u) = I$ iff I is preferred to G by at least two agents
- let $S = \{I^{1,2}, I^{1,3}, I^{2,3}, G^{1,2}, G^{1,3}, G^{2,3}\}$

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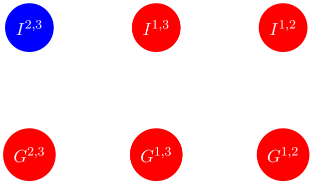
- let $S = \{I^{1,2}, I^{1,3}, I^{2,3}, G^{1,2}, G^{1,3}, G^{2,3}\}$
- let γ be such that, for each $i, j \in N$, and $s \in S$,

$$I^{i,j} \xrightarrow{\gamma} s = \{\{i\}, \{j\}\}$$

$$G^{i,j} \xrightarrow{\gamma} s = \{\{i\}, \{j\}\}$$

suppose the true preference profile is u^*, so $F(u^*) = I$



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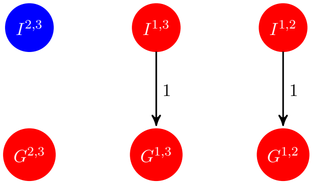

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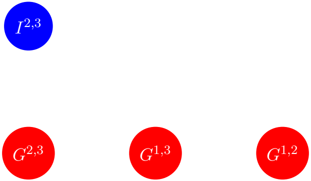

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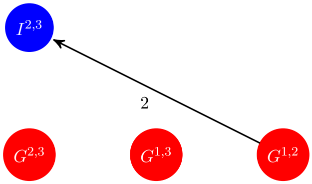

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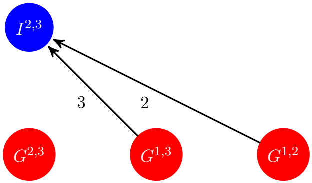

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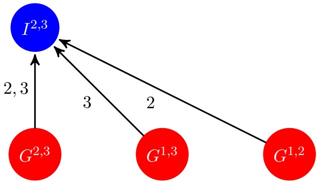

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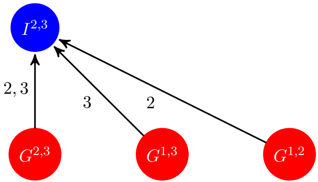

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a mechanism : (M, g)

$g : M \to A$

$F(u) = NE(M, g, u)$

Γ-implementation:

a rights structure $\Gamma = (S, h, \gamma)$

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let $S = M$ and $h = g$

$$s \xrightarrow{\gamma} t = \begin{cases} \{i\} & \text{if } t = (t_i, s_{-i}) \\ \emptyset & \text{otherwise} \end{cases}$$

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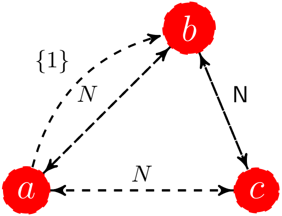
PwL is Γ-implementable, let $S = A$ and h be the identity map, design γ s.t. for each $x \neq a$ and $y \in A$,

- $x \xrightarrow{\gamma} y = \{N\}$
- $a \xrightarrow{\gamma} b = \{\{1\}, N\}$



PwL is not Nash implementable

let u and u' be as follows, where $F(u) = \{b, c\}$ and $F(u') = \{b\}$.

u

1	2	3
b	a	a
c	c	c
a	b	b

u'

1	2	3
b	a	a
a	c	c
c	b	b

outline

1 Γ-implementation

2 Γ-implementation with external stability

3 Γ underlying Nash implementability

4 γ-implementation

5 Γ-implementation with a minimal state space

Γ-implementation

- Suppose that alternative a is acceptable at u^1 according to the SCR in question. Then, if a does not fall in anyone's ranking relative to any other alternative in going from profile u^1 to profile u^2, monotonicity requires that a also be acceptable at u^2.

monotonicity and implementation

- Suppose that alternative a is acceptable at u^1 according to the SCR in question. Then, if a does not fall in anyone's ranking relative to any other alternative in going from profile u^1 to profile u^2, monotonicity requires that a also be acceptable at u^2.
- However, if a does fall relative to some b in someone's ranking, monotonicity imposes no restriction

monotonicity and implementation

F is monotonic if for each $u^1, u^2 \in \mathcal{P}$, and $a \in F(u^1)$, we have $a \in F(u^2)$ whenever for every $i \in N$,

$$\{b \in A : a\ u_i^1\ b\} \ \subseteq\ \{c \in A : a\ u_i^2\ c\}$$

monotonicity and implementation

- for an SCR F let $I(F) = \{a \in A : a \in F(u) \text{ for some } u\}$

F is image monotonic if for each $u^1, u^2 \in \mathcal{P}$, and $a \in F(u^1)$, we have $a \in F(u^2)$ whenever for every $i \in N$,

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Proposition: *Given an SCR F, the following are equivalent,*

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ii. F is $\Gamma - implementable$

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ii. F is $\Gamma - implementable$

iii. F is $\Gamma - implementable$ via an IB rights structure.

direct rights structures

a direct rights structure Γ^d is a triple (S^d, h^d, γ^d) where;

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- for each $(a, u) \in S^d$, $h^d(a, u) = a$
- for each $(a, u), (b, v) \in S^d$, and $i \in N$,

$$\{i\} \in (a, u) \xrightarrow{\gamma^d} (b, v) \text{ iff } a\ u_i\ b$$

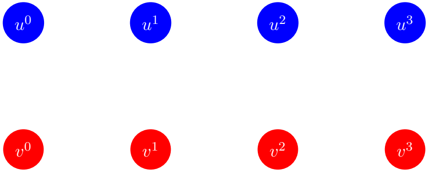
example: majority rule

- let $N = \{1, 2, 3\}$ and $A = \{a, b\}$
- there are four profiles $\{u^0, u^3, u^2, u^1\}$ where a is chosen since a is preferred to b by N, $\{1, 2\}$, $\{1, 3\}$, and $\{2, 3\}$ respectively
- similarly let $\{v^0, v^3, v^2, v^1\}$ be the profiles where the roles of a and b are changed, so b is chosen

consider the direct rights structure Γ^d tailored for this rule, where

$$S^d = \{(a, u^m), (b, v^m)\}_{m=1}^4$$

suppose the true preference profile is u^1, so $F(u^1) = a$



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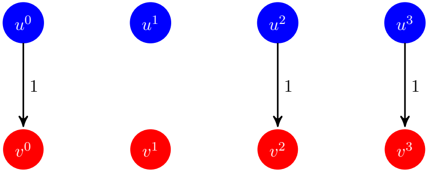

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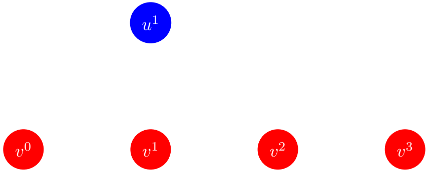

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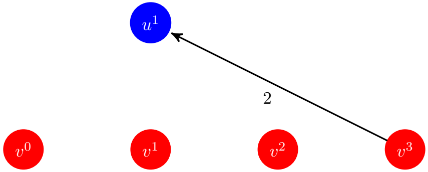

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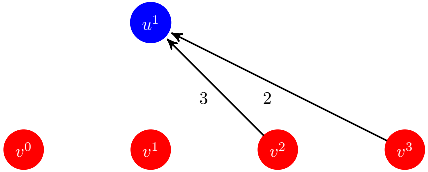

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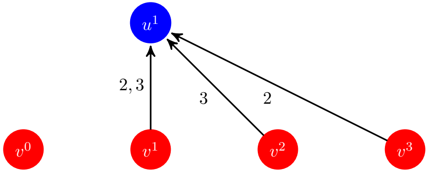

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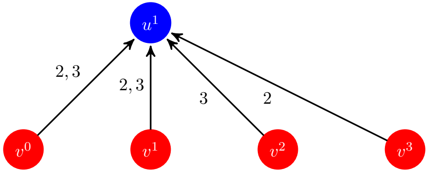

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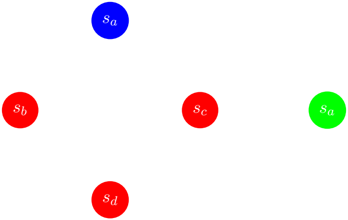
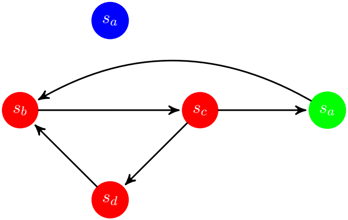
External stability of rights structures

Consider the rights structure Γ at u, where



blue: equilibrium states
red: neq-states with unacceptable outcomes
green: neq-states with acceptable outcomes

In general, there may not be a path connecting a non-equilibrium state to an equilibrium state



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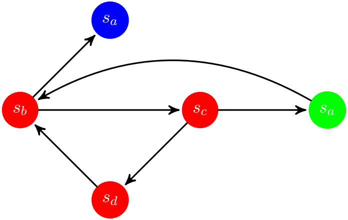
If the Γ-implementable SCR F is also unanimous, then always there exists such a path



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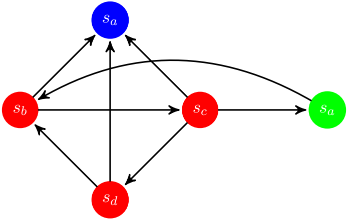
a rights structure, $\Gamma = (S, h, \gamma)$ *is externally stable if for each* $u \in \mathcal{P}$*, and for each* $s \in S$ *with an unacceptable outcome, there exists* $t \in E(\Gamma, u)$ *such that,* $(s \xrightarrow{\gamma} t) \cap (s \xrightarrow{u} t) \neq \emptyset$



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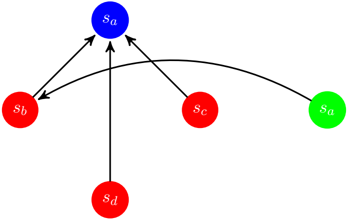

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Proposition: *F is implementable via an externally stable rights structure if and only if F is winner monotonic.*

Rights structures underlying the Nash implementability

Γ underlying the Nash implementability

for a rights structure $\Gamma = (S, h, \gamma)$,

(IB) Γ is individual based if for each distinct $s, t \in S$, $s \xrightarrow{\gamma} t$ is either empty or consists of only single agents.

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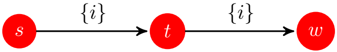
if $\{i\} \in (s \xrightarrow{\gamma} t) \cap (t \xrightarrow{\gamma} w)$, then



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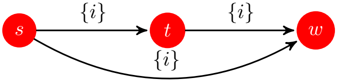

Γ underlying the Nash implementability

Proposition: *Given $N \geq 3$, F is Nash-implementable iff F is implementable via an individual based and individually transitive rights structure, Γ.*

$(\Leftarrow)$ given (S, h, γ), each $i \in N$, announces a state, an agent and a positive integer

let outcome function g be s.t. for each joint strategy m,

(1) if everybody announces the same state s, then we have $g(m) = h(s)$.

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(2) if everybody other than i announces the state s and i announces a state s'. If $\{i\} \in s \xrightarrow{\gamma} s'$, then $g(m) = h(s')$. If $\{i\} \notin s \xrightarrow{\gamma} s'$, then $g(m) = h(s)$.

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(3) if at least three different states are announced, then let i be the agent who announces the highest integer.
Let agent j be the agent whom is indicated by agent i.

Let s'' is the state announced by agent j. We have, $g(m) = h(s'')$.

dc-mechanism

design object is a dc-mechanism $(M, \mathcal{D}, g)$

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an outcome function $g : M \rightarrow A$

for each agent i, $\mathcal{D}_i$ maps each joint strategy m to a subset of M_i, i.e. $\mathcal{D}_i : M \rightarrow M_i$.

In a dc-mechanism, if an agent i would deviate from strategy m, he is constraint to choose his strategy from $\mathcal{D}_i(m)$.

a joint strategy m is an equilibrium of $(M, \mathcal{D}, g)$ at u iff for each $i \in N$ and $m_i' \in \mathcal{D}_i(m)$,

$$g(m)\ u_i\ g(m_i', m_{-i})$$

we denote the equilibria of $(M, \mathcal{D}, g)$ at u, by $\mathbf{E}(M, \mathcal{D}, g, u)$.

Proposition: *Given $N \geq 3$, F is Γ-implementable if and only if there exists a dc-mechanism, $(M, \mathcal{D}, g)$, such that for each u, $F(u) = \mathbf{E}(M, \mathcal{D}, g, u)$.*

*A SOCIAL EQUILIBRIUM EXISTENCE THEOREM**

BY GERARD DEBREU

COWLES COMMISSION FOR RESEARCH IN ECONOMICS

Communicated by J. von Neumann, August 1, 1952

In a wide class of social systems each agent has a range of actions among which he selects one. His choice is not, however, entirely free and the actions of all the other agents determine the subset to which his selection is restricted. Once the action of every agent is given, the outcome of the social activity is known. The preferences of each agent yield *his* complete ordering of the outcomes and each one of them tries by choosing his action in his restricting subset to bring about the best outcome according to his

γ-implementation

γ-implementation

we specify $S = A$ and h is the identity map

Definition: For each u, an alternative a is a γ-equilibrium at u if

$$\text{for each } b \in A,\ (a \xrightarrow{\gamma} b) \cap (a \xrightarrow{u} b) = \emptyset$$

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Definition: F is γ- implementable if there is a code of rights, γ, such that

$$\text{for each } u \in \mathcal{P},\ F(u) = E(\gamma, u)$$

example: Walrasian equilibrium

- Consider any 2×2 pure exchange economy where agents have monotonic, continuous and strictly convex preferences over the entire consumption space R^2_+
- Given an endowment vector $\omega >> 0$, let F^ω be the rule that chooses the Walrasian allocations at each preference profile.

for given ω, for each $a, b \in R^2_+$, let

$$a \xrightarrow{\gamma} b = \begin{cases} \{1\}, \{2\} & \text{if } a, b \text{ and } \omega \text{ are collinear or } a \text{ is not feasible,} \\ \emptyset & \text{otherwise} \end{cases}$$

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Proposition: *F is γ-implementable if and only if F satisfies Maskin monotonicity and binary consistency.*

binary consistency

Consider any $u \in \mathcal{P}$ and $a, b \in A$ such that b is not Pareto dominated by a at u. Let $\mathbf{u^{ab}}$ stand for the profile obtained from u such that the only difference is for each $c \in A \setminus \{a, b\}$ and $i \in N$, we have $a \; u_i^{ab} \; c$ and $b \; u_i^{ab} \; c$.

u

1	2	3
c	c	c
a	a	b
b	b	a

.

u^{ab}

1	2	3
a	a	b
b	b	a
c	c	c

γ-implementation

F satisfies binary consistency if for each $u \in \mathcal{P}$ and $a \in A$, we have $a \in F(u)$ whenever for each $b \in A$ that is not Pareto dominated by a at u, one has $a \in F(u^{ab})$.

Proposition: *F is γ-implementable if and only if F satisfies Maskin monotonicity and binary consistency.*

State complexity of rights structures

critical rights structures

a critical rights structure Γ^c is a triple (S^c, h^c, γ^c) where;

$$S^c = \{(a, u) : \ u \in \Lambda(F, a)\}$$

- for each $(a, u) \in S^c$, $h^c(a, u) = a$
- for each $(a, u), (b, v) \in S^c$, a coalition $K \in (a, u) \xrightarrow{\gamma^c} (b, v)$ iff there exists $i \in K$ s.t. $a \ u_i \ b$

critical profile

- for each $a \in A$, a preference profile u is critical for a, if for any b, any agent i with $a \; u_i \; b$ reverses his preference between a and b for any b , then F no longer chooses a in the new profile

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Ex: Consider the majority rule and let u and u' be as follows,

u

1	2	3
a	a	c
c	c	b
b	b	a

.

u'

1	2	3
a	a	c
c	b	a
b	c	b

- u is critical whereas u' is not

state complexity of rights structures

Proposition: *Given* (S, h)*, if an IB rights structure* (S, h, γ) *implements* F*, then the number of states in* S *is at least equal to the number of states in* S^c*, i.e.* $|S| \geq |S^c|$*.*

related literature

rights structures

- Sen (1960)
- Sertel (2001)
- Ben and Sugden (2011)

effectivity fncs. and imp.

- Moulin and Peleg (1982)
- Peleg and et. al.

dc-mechanisms

- Debreu (1952)
- Glazer and Rubinstein (2012)

external stability

- Greenberg (1990)
- Chwe (1994)

conclusion

Our analysis of Γ-implementation shows that,

- we can implement SCRs by explicitly designing simple rights structures
- why complicated game form are inevitable in classical implementation.