Choice with Affirmative Action

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joint with...

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Question: How to reconcile these two potentially conflicting objectives?

examples from applications

Priority orderings	*Minorities*
exam-score order in school choice	neighborhood students

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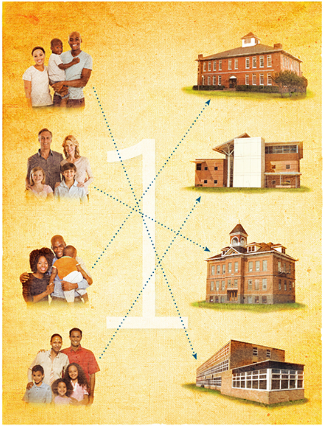
a merit order in job assignment	applicants with disabilities
time-of-application in visa assignment	highly educated applicants

school choice application

- A well-known example is the school choice problem in which each school has a certain number of seats (a capacity) to be allocated among students.



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- Although student preferences are elicited from the students, endowing each school with a choice rule is a part of the design process.
- When there are affirmative action concerns, which choice rule to use is non-trivial.
- Recent emprical studies, such as Leashno & Lo (2021), find that the design of the choice rule can have larger welfare implications than designing the rest of the mechanism.

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Outline

Part 1: The new framework and the ‘comparative statics’ axioms.

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Part 2: Relationship to the choice rules in practice.

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A choice rule C for a school with q available seats (capacity) maps each problem $(S, \tau, \succ)$ to a nonempty subset $C(S, \tau, \succ) \subseteq S$ without exceeding the capacity, i.e. $|C(S)| \leq q$.

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- **Priority-compatibility:** A student is chosen over a higher priority student only if the former student is a minority student and the latter is a majority student.
- **Substitutability:** A chosen student remains chosen when the set of students shrinks, everything else the same.

Substitutability is crucial because...

- Substitutable choice rules have been a standard tool following the seminal work of Kelso and Crawford, 1982, in broadening the classical matching model with single priority.
- Hatfield and Milgrom, 2005 show that substitutability guarantees the existence of stable matchings.
- Hatfield and Kojima, 2006 show that substitutability is almost necessary for the non-emptiness of the core in allocations problems
- Similarly, several classical results of matching literature have been generalized with substitutable choice rules (Roth and Sotomayor, 1990; Hatfield and Milgrom, 2005).

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priority improvements

- Given $(S, \tau, \succ)$, a priority ordering $\succ'$ is an improvement over $\succ$ for a student s, if when we move from $\succ$ to $\succ'$, the priority order of s weakly improves relative to each other student and strictly improves relative to at least one student, while the priority relation within other students stays the same.

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$$
\begin{array}{ccc}
\underline{(S, \tau, \succ)} & & \underline{(S, \tau, \succ')} \\
1 & & 3 \\
2 & & 1 \\
3 & \longrightarrow & 2 \\
4 & & 4 \\
5 & & 5
\end{array}
$$

type changes

- Given $(S, \tau, \succ)$, a type function τ' is obtained from τ by changing the type of a student s if $\tau(s) = 1$ and $\tau'(s) = 0$; and for each $s' \in S \setminus \{s\}$, $\tau(s) = \tau'(s)$.

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Theorem 1: *A choice rule is an affirmative action rule that satisfies the responsiveness axioms if and only if it admits a responsive reserve representation.*

reserve representations

- Reserve representation: For a given capacity q, at each problem, in addition to all the top-q ranked (top-chunk) minority students, a certain number of non–top-q ranked (bottom-chunk) minority students, called a reserve number, are chosen.

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example

Let $|\mathcal{S}| = 6$ and $q = 4$. Consider the choice rule C that admits a reserve representation via the following reserve function R that maps each type configuration of the top-q ranked students, denoted by ρ, to a reserve number.

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i. if ρ does not include any minority student, then $R(\rho) = 2$,

$(S, \tau, \succ)$
1
2
3
4
5
6

$$C(S, \tau, \succ) = \{1, 2, 5, 6\}$$

example

ii. if ρ includes one minority student, say 3, then $R(\rho) = 1$,

$$\begin{array}{c} \underline{(S,\tau,\succ)} \\ 1 \\ 2 \\ \color{red}{3} \\ 4 \\ \hline \color{red}{5} \\ \color{red}{6} \end{array}$$

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iii. if ρ includes two minority students, say m_1 and m_2 such that $m_1 \; \rho \; m_2$, then $R(\rho) = 0$ if $rank(m_1, \rho) \neq 1$ and $rank(m_2, \rho) = 4$; and $R(\rho) = 1$ otherwise,

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iv. if ρ includes three or four minority students, $R(\rho) = 0$.

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Let $|\mathcal{S}| = 6$ and $q = 4$. Consider the choice rule C that admits a reserve representation via the following reserve function R that maps each type configuration of the top-q ranked students, denoted by ρ, to a reserve number.

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ii. if ρ includes one minority student, say m, then $R(\rho) = 1$,

iii. if ρ includes two minority students, say m_1 and m_2 such that $m_1 \; \rho \; m_2$, then $R(\rho) = 0$ if $rank(m_1, \rho) \neq 1$ and $rank(m_2, \rho) = 4$; and $R(\rho) = 1$ otherwise,

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Negative responsiveness axioms

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- **Negative responsiveness to type changes (NRTC):** If the type of a chosen minority student is changed, then all the majority students who used to be chosen should still be chosen.

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Choice rules in practice

Lexicographic affirmative action rules

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- school choice rules in Boston and in Chicago
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Although these applications include different institutional constraints, the lexicographic feature remains common.

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1 & & 3 \\
2 & & 5 \\
3 & \longrightarrow & 1 \\
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\end{array}
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an example: no reserves

- Let $|\mathcal{S}| = 5$ and $q = 3$. For $l = [$open, open, open$]$

$$\begin{array}{c|ccc} \underline{(S,\tau,\succ)} & \underline{\succeq} & \underline{\succeq} & \underline{\succeq} \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 \end{array} \quad \longrightarrow \quad C^l(S,\tau,\succ) = \{1,2,3\}$$

an example: reserve first

- Let $|\mathcal{S}| = 5$ and $q = 3$. For $l = [$reserve, open, open $]$

$$
\begin{array}{c|ccc}
\underline{(S,\tau,\succ)} & \underline{\succ^a} & \underline{\succ} & \underline{\succ} \\
1 & 1 & 1 & 1 \\
2 & 5 & 2 & 2 \\
3 & 2 & 3 & 3 \\
4 & 3 & 4 & 4 \\
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\end{array}
\quad \longrightarrow \quad C^l(S,\tau,\succ) = \{1,2,3\}
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an example: reserve last

- Let $|\mathcal{S}| = 5$ and $q = 3$. For $l =$ [open, open, reserve]

$$\begin{array}{c|ccc}
\underline{(S,\tau,\succ)} & \underline{\succ} & \underline{\succ} & \underline{\succ^a} \\
1 & 1 & 1 & 1 \\
2 & 2 & 2 & 5 \\
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A choice rule admits a lexicographic representation if there exists a lexicographic order, such that for each problem $(S, \tau, \succ)$, $C(S, \tau, \succ)$ is obtainable via the associated lexicographic procedure.

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Two special cases are:

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An axiomatization of lexicographic choice

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effective type changes

- Given a problem $(S, \tau, \succ)$ and a chosen minority s, changing the type of s is effective if it results in new chosen minority students, i.e. $C^m(S, \tau', \succ) \setminus C^m(S, \tau, \succ) \neq \emptyset$ where τ' is obtained from τ by changing the type of s.

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$(S, \tau, \succ)$		$(S, \tau', \succ)$
1		1
2		2
3	$\longrightarrow$	3
4		4
5		5

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$$
\begin{array}{c}
\underline{(S, \tau, \succ)} \\
1 \\
2 \\
3 \\
4 \\
5 \\
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\end{array}
\quad \longrightarrow \quad
\begin{array}{c}
\underline{(S, \tau', \succ)} \\
1 \\
2 \\
3 \\
4 \\
5 \\
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\end{array}
$$

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Theorem 3: *A choice rule is an affirmative action rule that satisfies the responsiveness and the monotonicity axioms if and only if it admits a lexicographic representation.*

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summary

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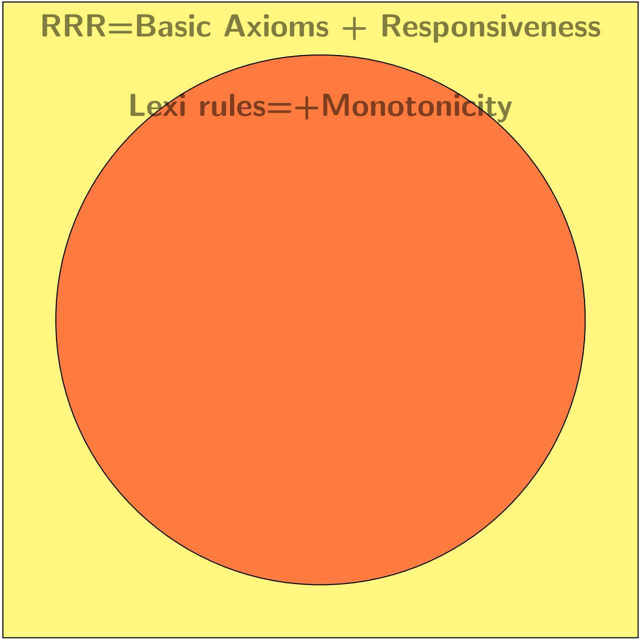
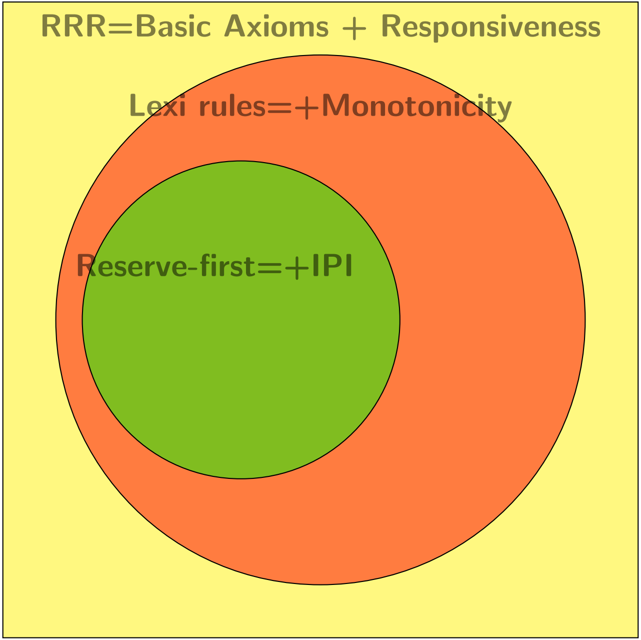
RRR=Basic Axioms + Responsiveness

summary



summary


RRR=Basic Axioms + Responsiveness
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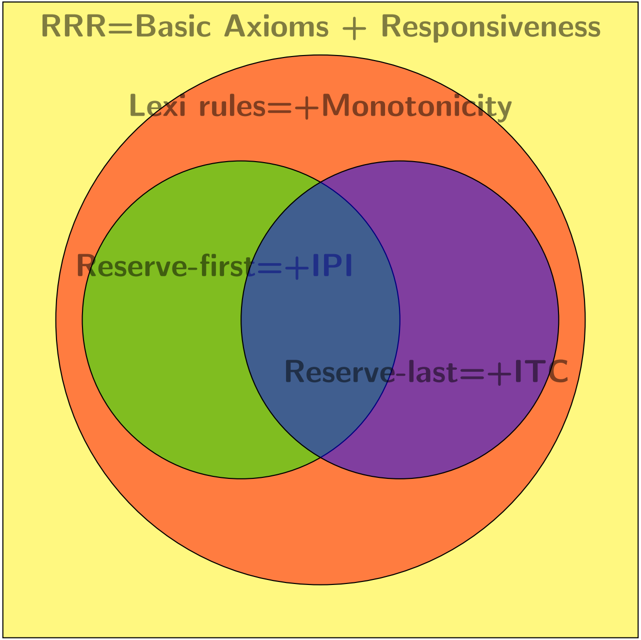
summary


RRR=Basic Axioms + Responsiveness
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conclusion: Thank you !

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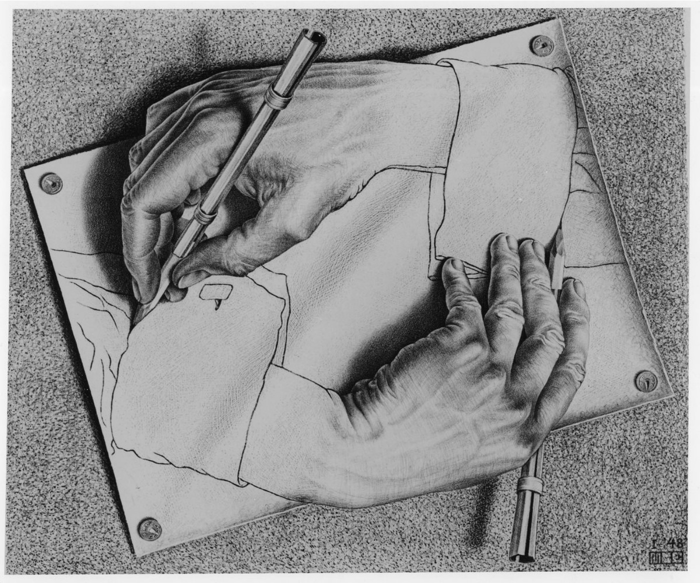

Thank you!

Responsiveness axioms

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conclusion

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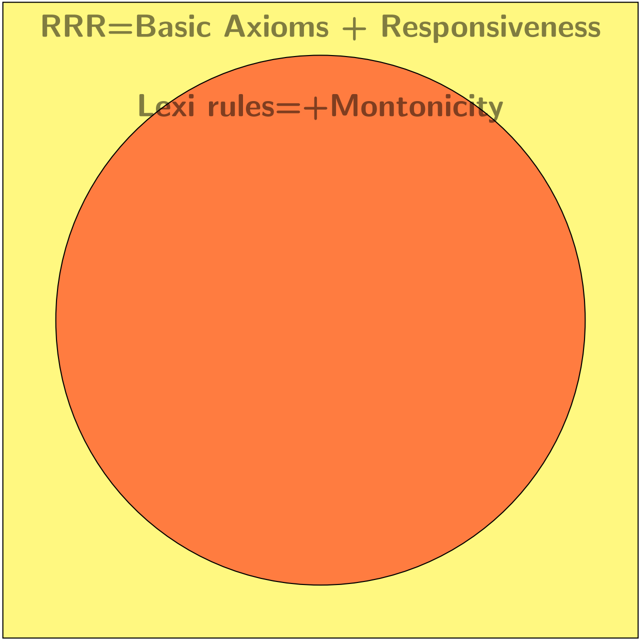
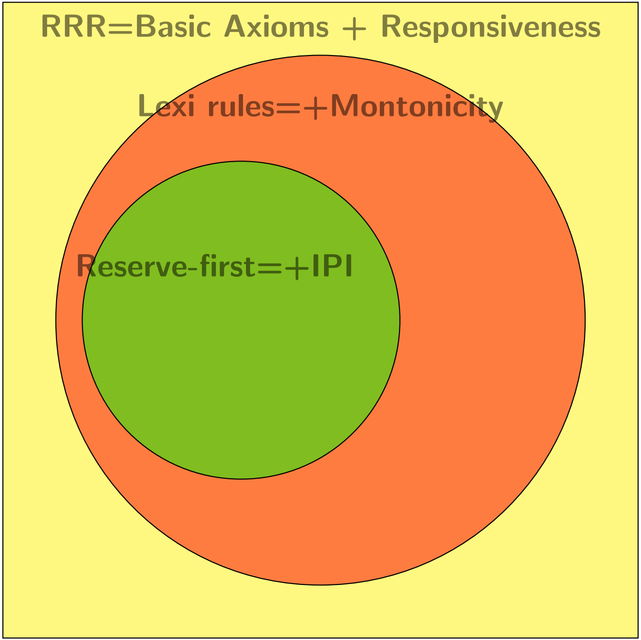
conclusion



conclusion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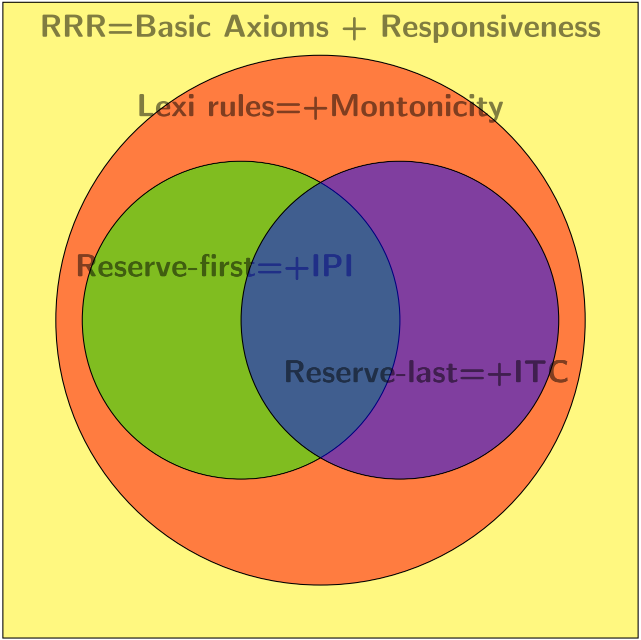
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Threshold-reserve representation

- **Conditional invariance under priority improvements (CIPI):** If changing the type of a chosen minority student is effective, then improving the priority of a lower ranked chosen minority student does not affect the choice unless he is moved above the former student.

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a characterization

Theorem 3: *A choice rule C is an affirmative action rule that satisfies conditional invariance under priority improvements if and only if it admits a threshold-reserve representation.*