The New York Times

SCIENCE

SHARE

Divide Your Rent Fairly

When you're sharing an apartment with roommates, it can be a challenge to decide who takes which bedroom, and at what price. Sit down with your roommates and use the calculator below to find the fair division. | RELATED ARTICLE

What's your total rent? $ 1000

How many of you are there? 2 3 **4** 5 6 7 8

Your names are:

A. Roommate A

B. Roommate B

C. Roommate C

D. Roommate D

And the rooms in your apartment are:

1. Room 1

2. Room 2

3. Room 3

4. Room 4

Start division »

Once you start, each roommate keeps choosing a preferred room at a certain price until a fair division is reached.

NYT: Sperner's lemma defeats the rental harmony problem

The New York Times

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To Divide the Rent, Start With a Triangle

By ALBERT SUN APRIL 28, 2014

Last year, two friends and I moved into a small three-bedroom apartment in Manhattan. We chose it for its relatively reasonable price — around $3,000 a month — and its convenient location. Just finding it was a challenge, but then we faced another one: deciding who would get each bedroom.

The bedrooms were different sizes, ranging from small to very small. Two faced north toward the street and had light; the third and smallest faced an alley. The largest had two windows; the midsize room opened onto the fire escape.

Every month, unrelated people move into apartments together to save on rent. Many decide to simply divide the rent evenly, or to base it on

Ashwin

Bret

Chad

NYT: Sperner's lemma defeats the rental harmony problem

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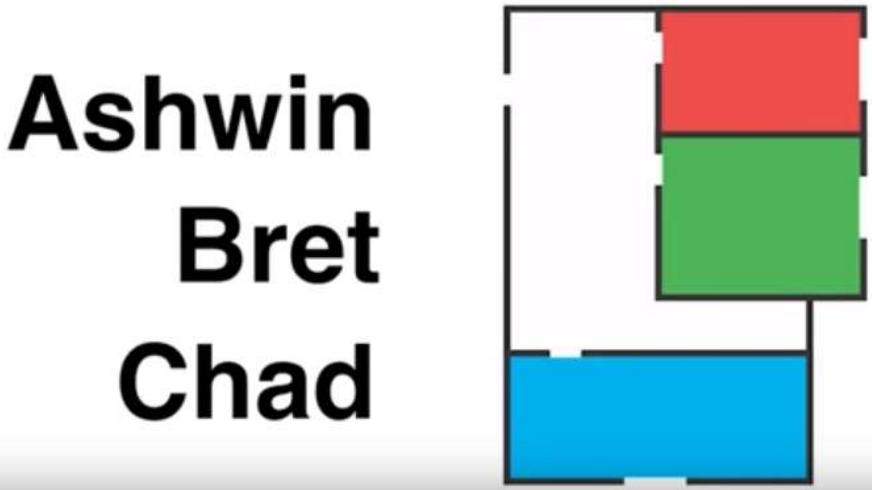
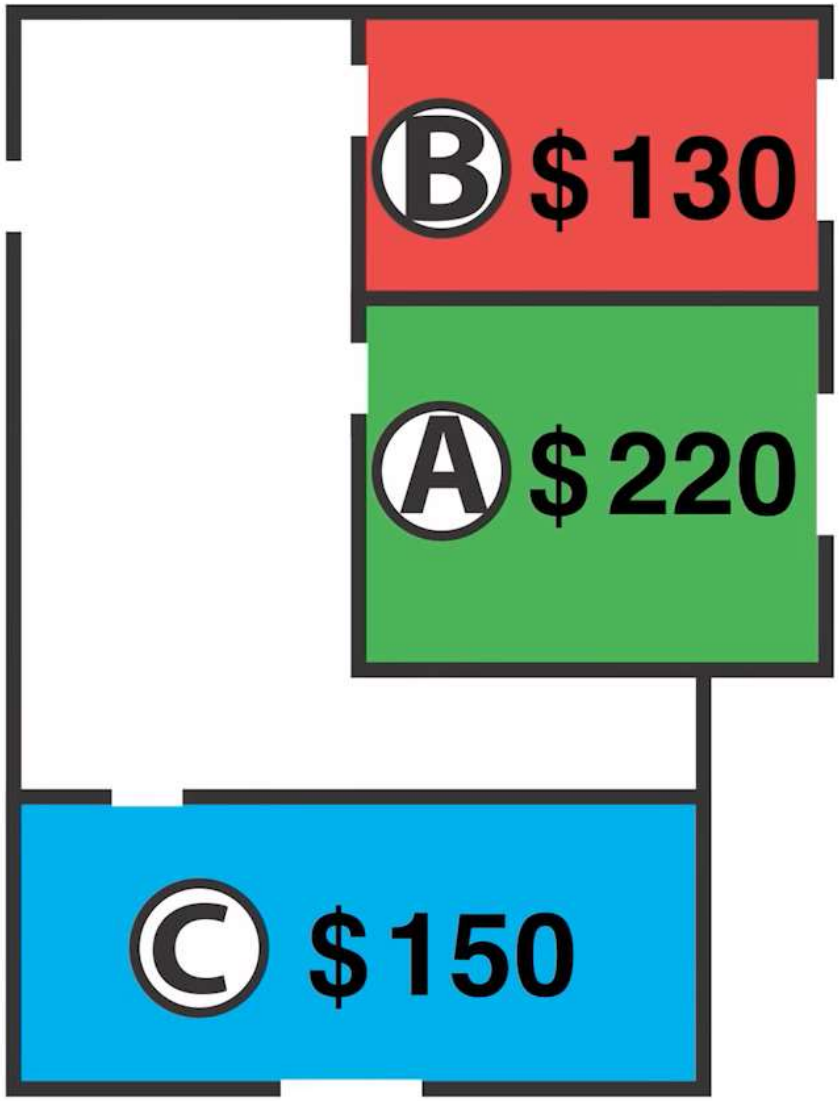
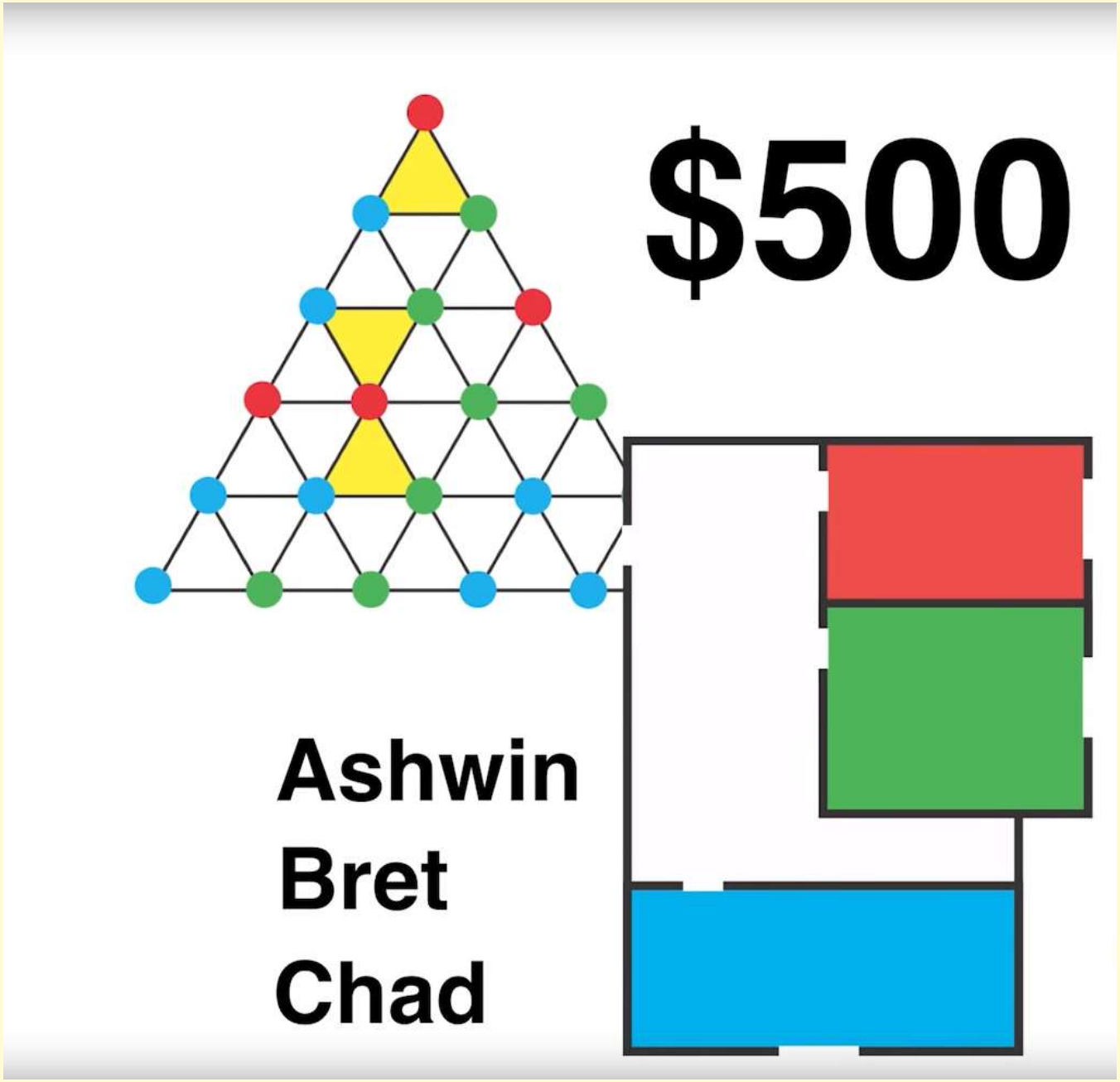


an allocation



a fair allocation???



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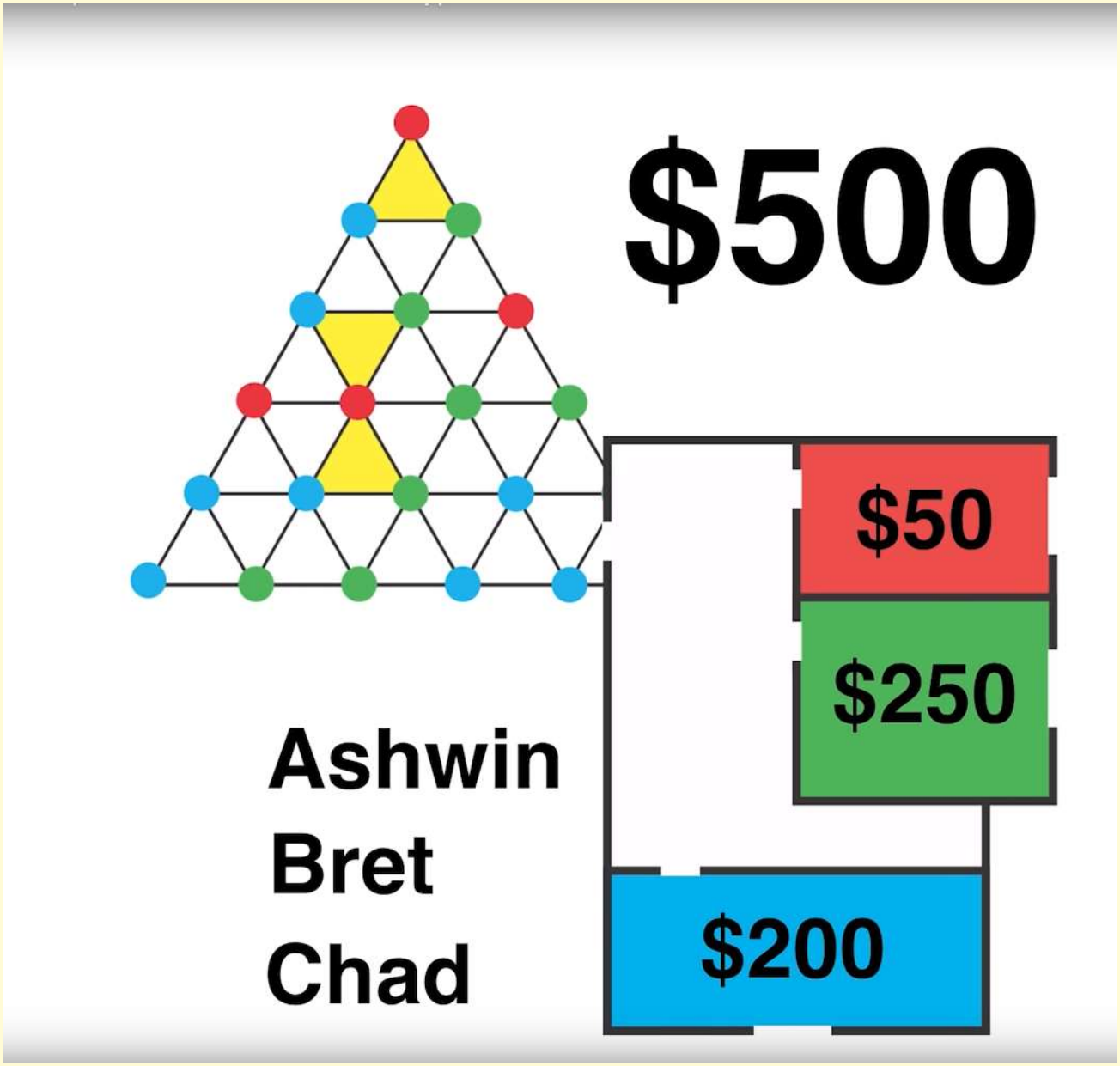

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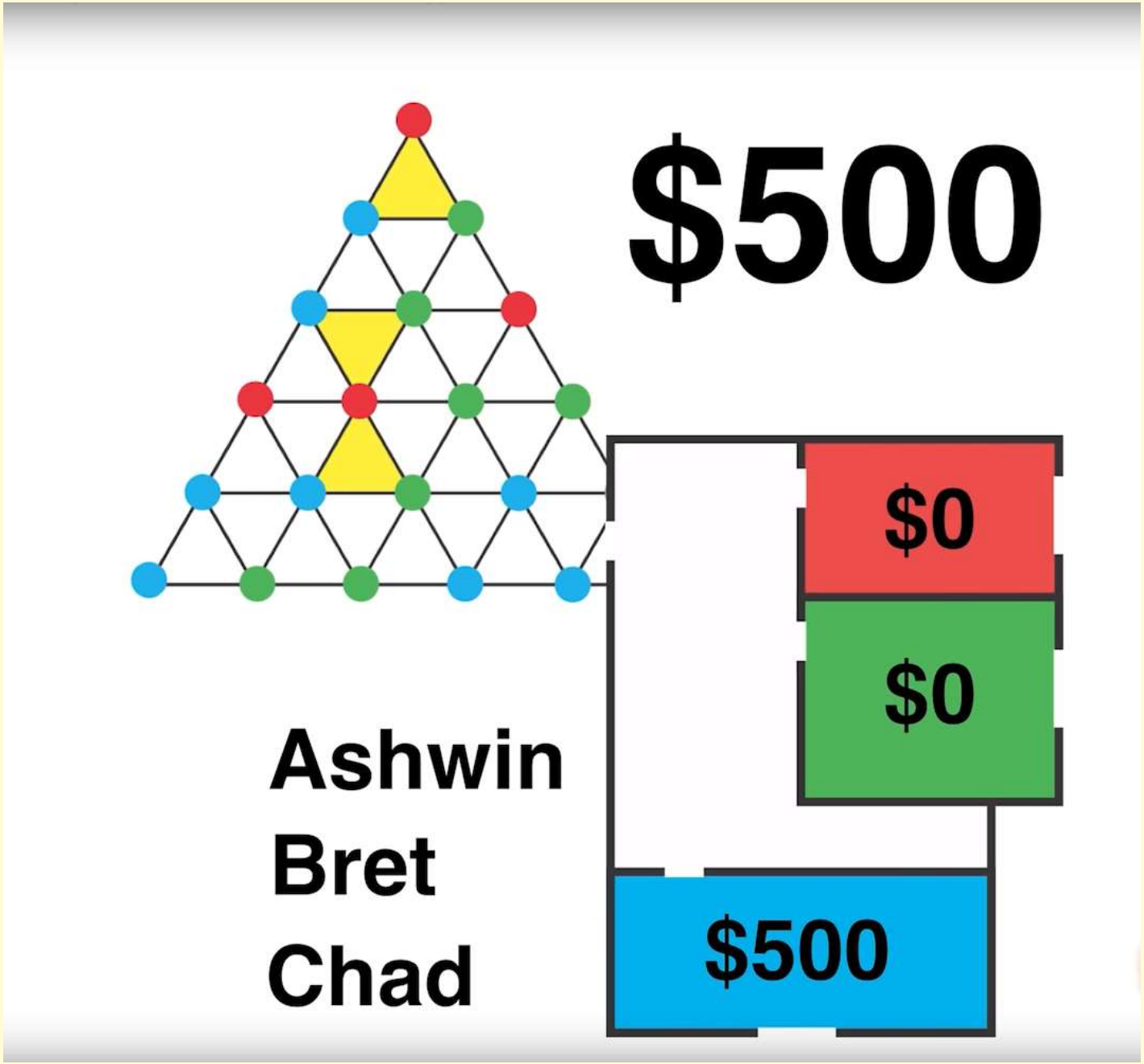

Rental Harmony A 228-IQ pts question

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Your Pet Contest.

BY MARILYN VOS SAVANT

Ask Marilyn

Five college girls will share a three-bedroom apartment with one parking space included in the monthly rent of $1425. Lynn will have her own private bedroom and use of the parking space. Of the other girls, only Susan needs a parking space, which will cost her an additional $50 monthly, with the space located a block away. What is a fair share of the rent for all the girls to pay?

Can you come up with a fair rent for five young women?

—A College Parent, McMurray, Pa.

The cost of Susan's extra parking space gives us a notion of what Lynn's parking space is worth: about $75 (it's much closer). Lynn should pay that herself. Subtracting $75 from $1425 gives us $1350 for the apartment rent alone.

I think Lynn should pay for two shares (for twice the space), and the other girls should pay for one share. Dividing $1350 by 6 equals $225 per share. So the other four each would pay $225 for rent, and Lynn would pay $450 (2 times $225) plus $75 for the parking space, a total of $525. (Note: Susan pays for her parking space separately.) And even so, Lynn is getting the best deal.

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Econometrica, Vol. 59, No. 4 (July, 1991), 1023–1039

FAIR ALLOCATION OF INDIVISIBLE GOODS AND CRITERIA OF JUSTICE

By Ahmet Alkan, Gabrielle Demange, and David Gale[1]

A set of n objects and an amount M of money is to be distributed among m people. Example: the objects are tasks and the money is compensation from a fixed budget. An elementary argument via constrained optimization shows that for M sufficiently large the set of efficient, envy free allocations is nonempty and has a nice structure. In particular, various criteria of justice lead to unique best fair allocations which are well behaved with respect to changes of M. This is in sharp contrast to the usual fair division theory with divisible goods.

Keywords: Fairness, envy-free, Pareto efficient, justice, Rawlsian justice.

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Keywords: Fairness, envy-free, Pareto efficient, justice, Rawlsian justice.

Amer. Math. Monthly, 106(1999), 930-942

RENTAL HARMONY: SPERNER'S LEMMA IN FAIR DIVISION

FRANCIS EDWARD SU

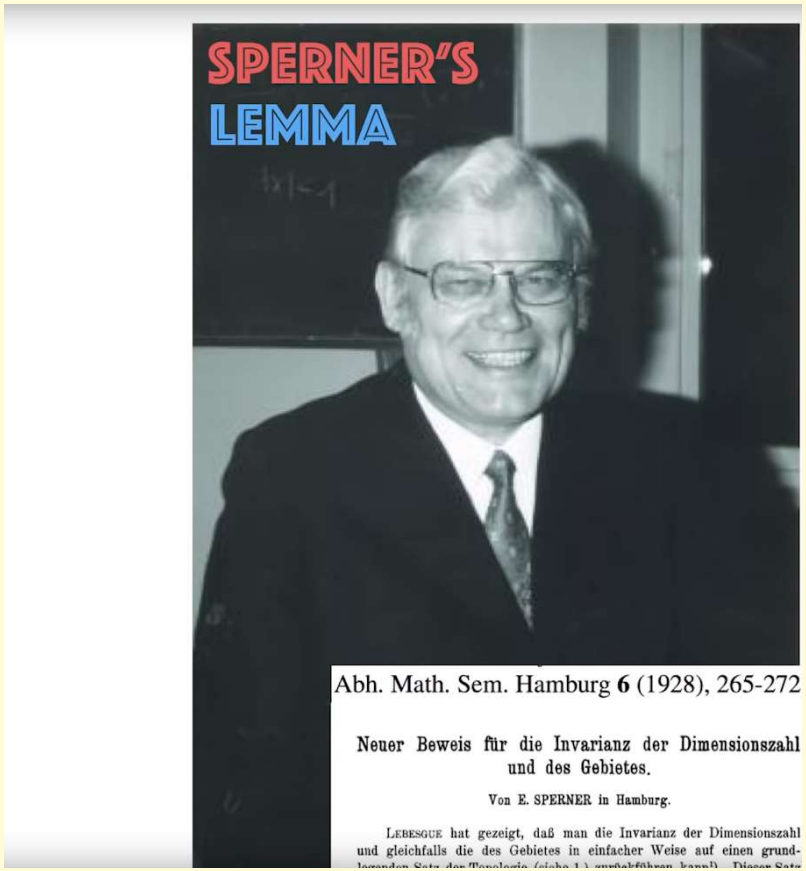
My friend's dilemma was a practical question that mathematics could answer, both elegantly and constructively. He and his housemates were moving to a house with rooms of various sizes and features, and were having trouble deciding who should get which room and for what part of the total rent. He asked, "Do you think there's always a way to partition the rent so that each person will prefer a different room?"

As we shall see, with mild assumptions, the answer is yes. This rent-partitioning problem is really a kind of *fair-division* question. It can be viewed as a generalization of the age-old *cake-cutting* problem, in which one seeks to divide a cake fairly among several people, and the *chore-division* problem, posed by Martin Gardner in [6, p. 124], in which one seeks to fairly divide an undesirable entity, such as a list of chores. Lately, there has been much interest in fair division (see, for example, the recent books [3] and [11]), and each of the related problems has been treated before (see [1], [4], [10]).

We wish to explain a powerful approach to fair-division questions that unifies these problems and provides new methods for achieving approximate *envy-free* divisions, in which each person feels she received the "best" share. This approach was carried out by Forest Simmons [13] for cake-cutting and depends on a simple combinatorial result known as Sperner's lemma. We show that the Sperner's lemma approach can be adapted to treat chore division and rent-partitioning as

THE AMERICAN MATHEMATICAL MONTHLY MAA

Sperner's Lemma



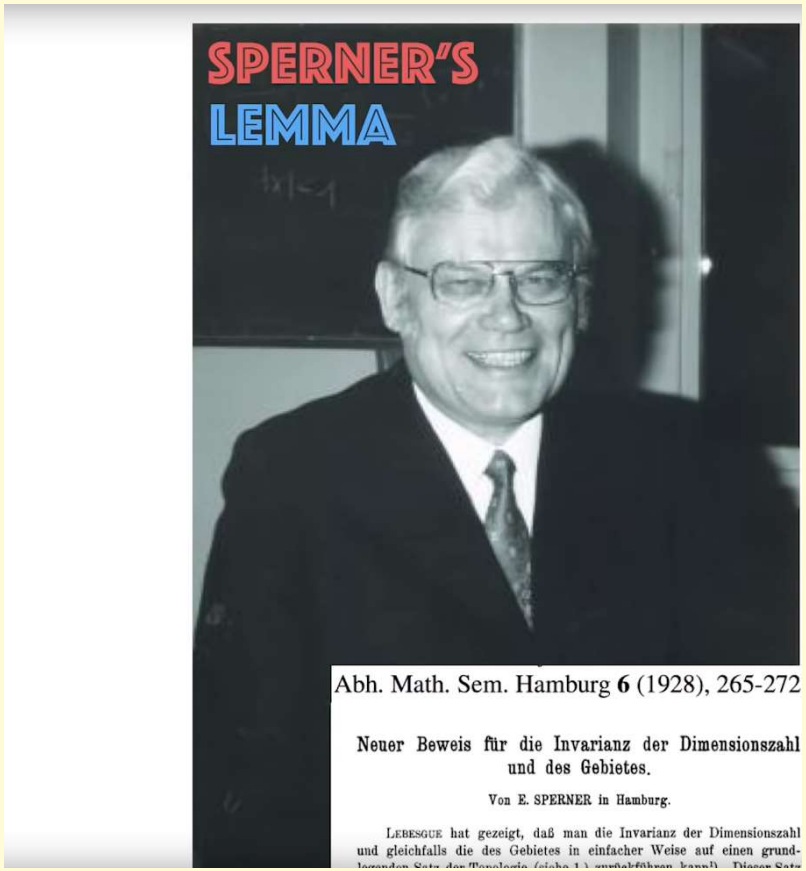
Abh. Math. Sem. Hamburg **6** (1928), 265-272

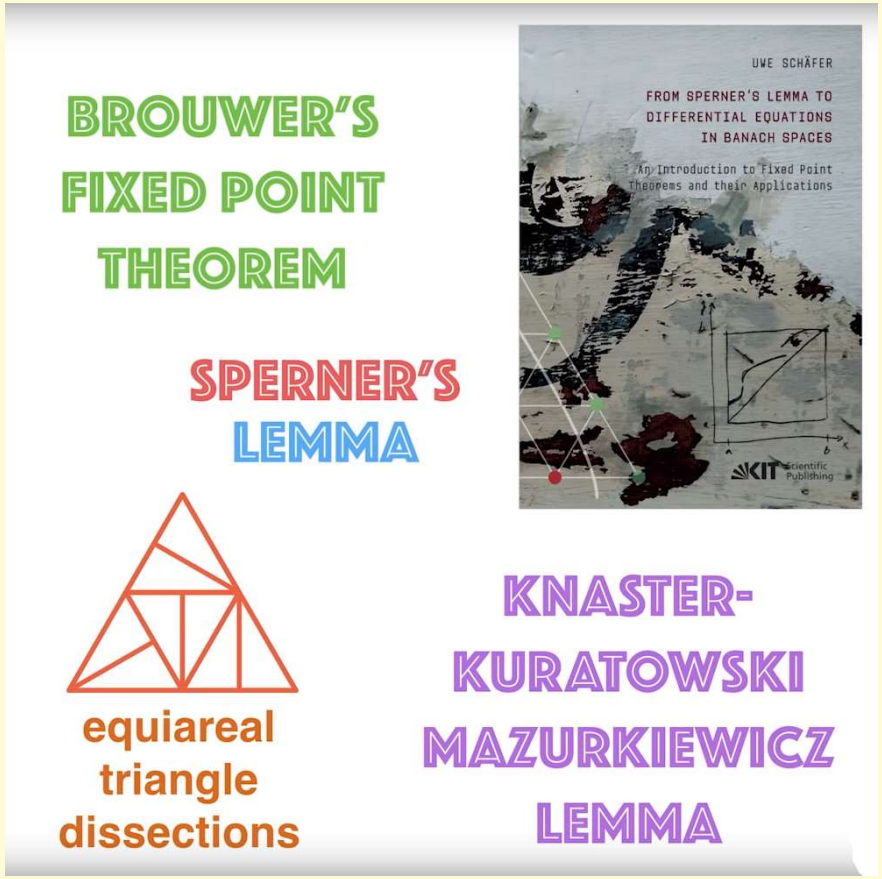
Neuer Beweis für die Invarianz der Dimensionszahl und des Gebietes.

Von E. SPERNER in Hamburg.

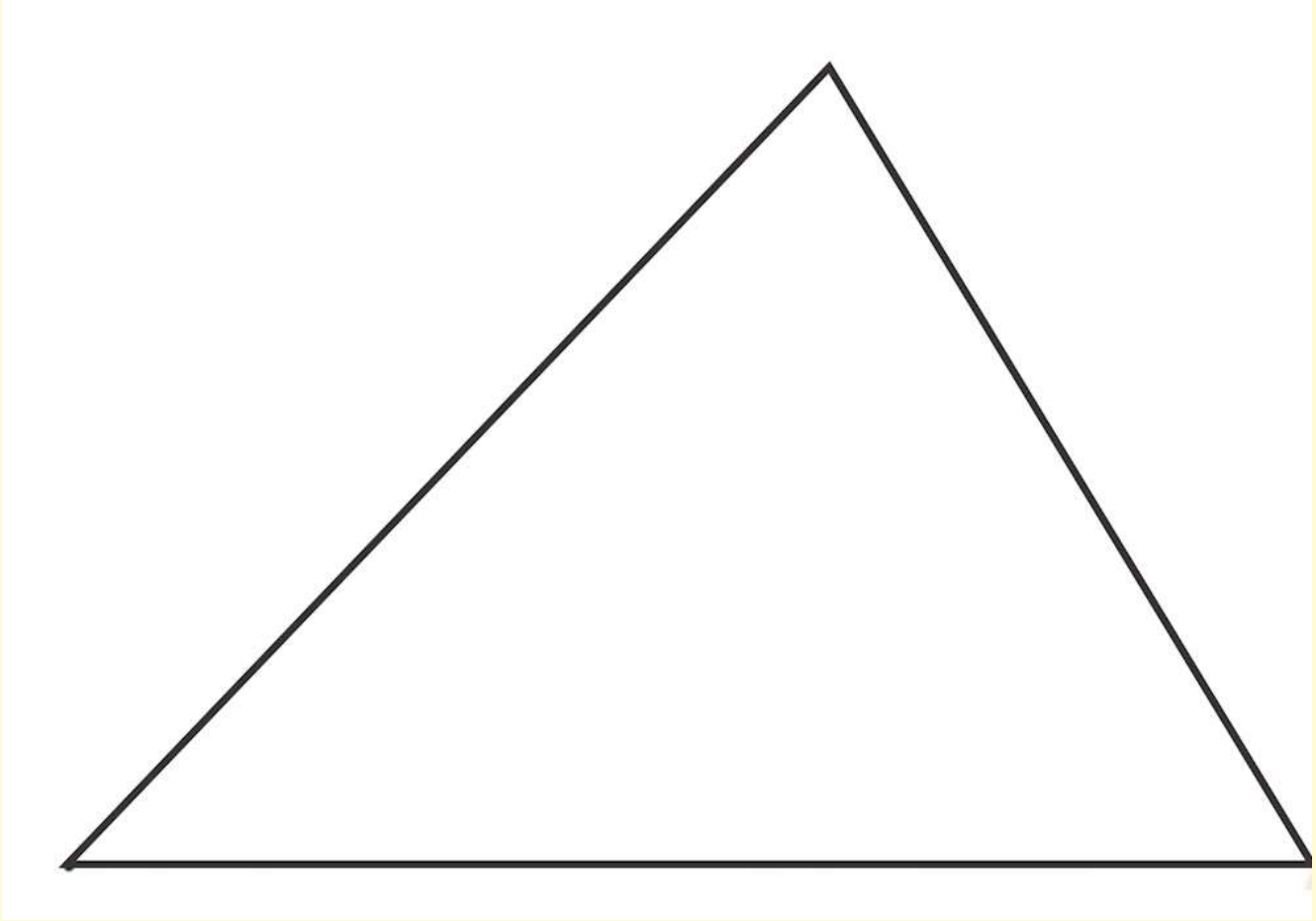
LEBESGUE hat gezeigt, daß man die Invarianz der Dimensionszahl und gleichfalls die des Gebietes in einfacher Weise auf einen grund-

Sperner's Lemma

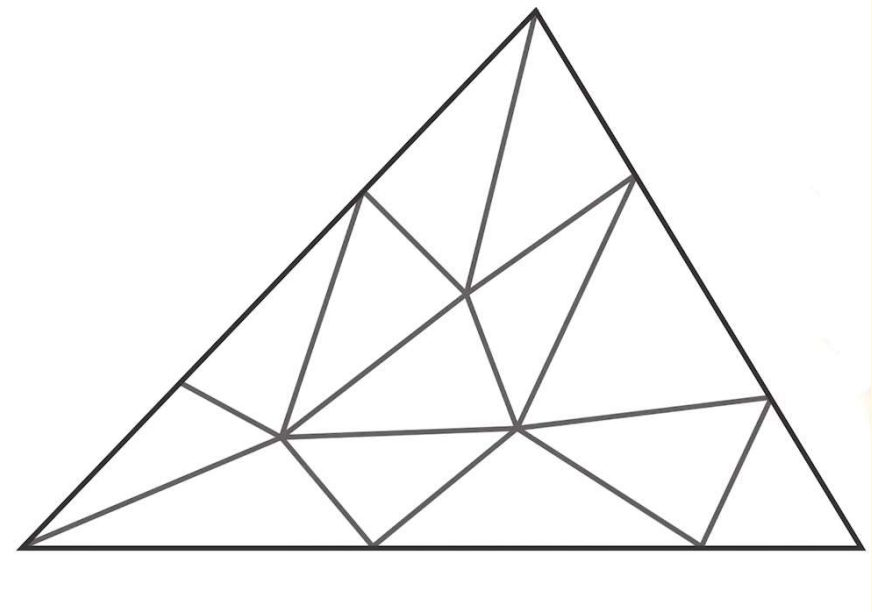




Statement of SL: first, take a triangle



triangulation: partition it into sub-triangles



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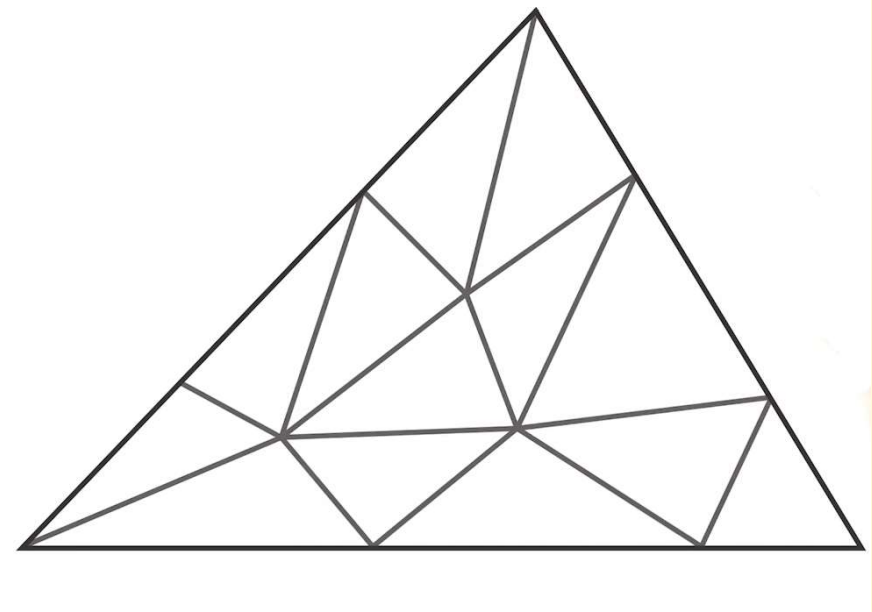
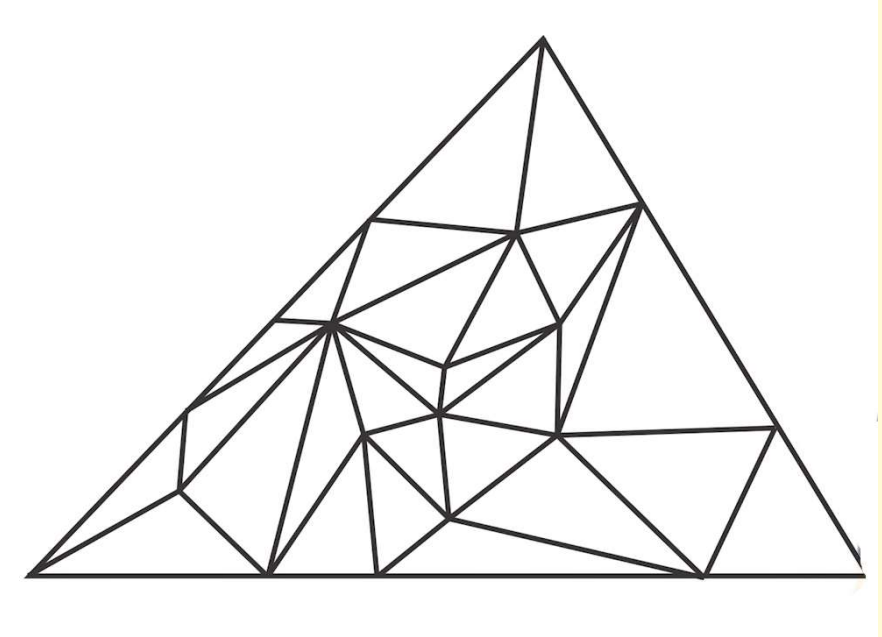
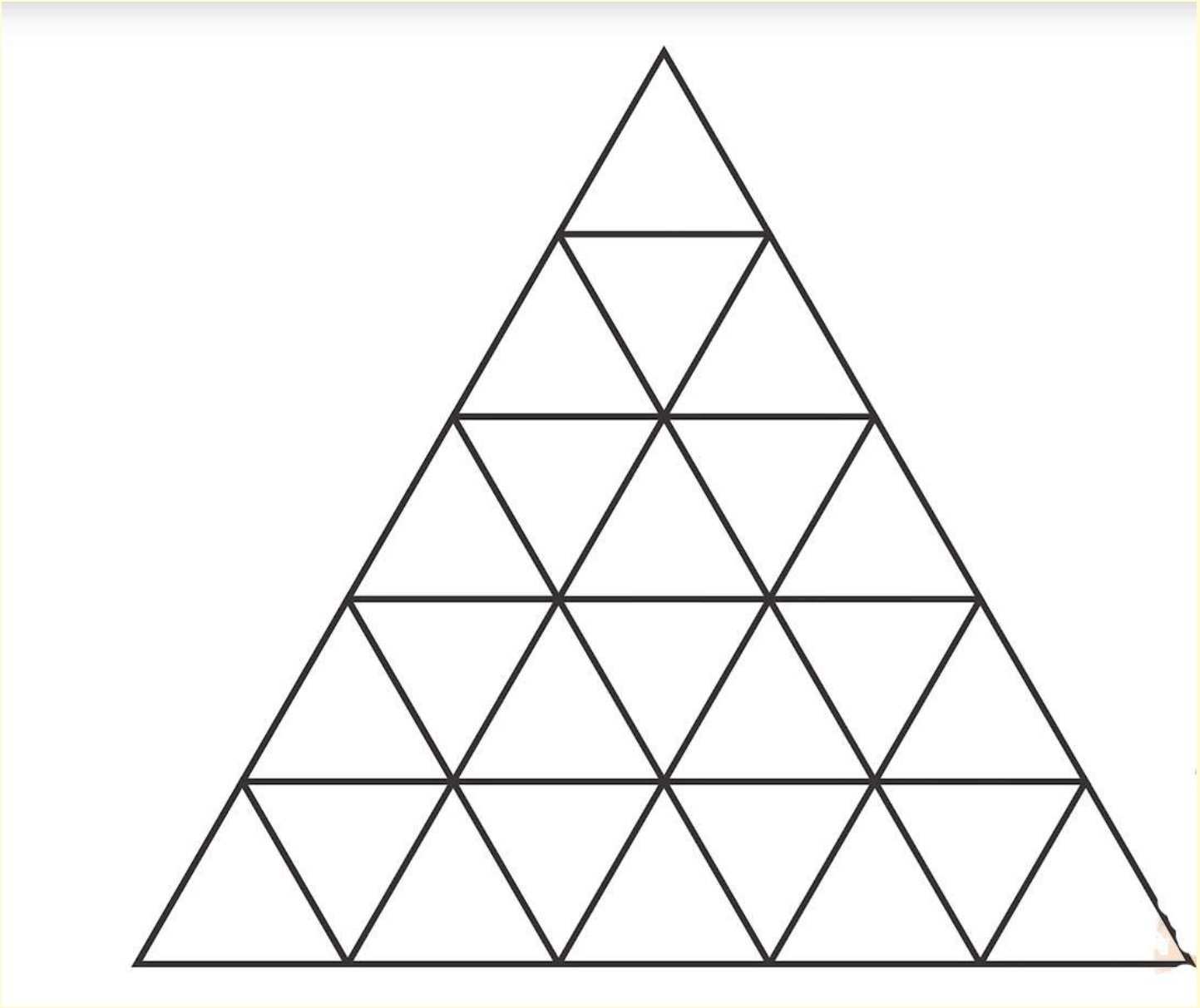
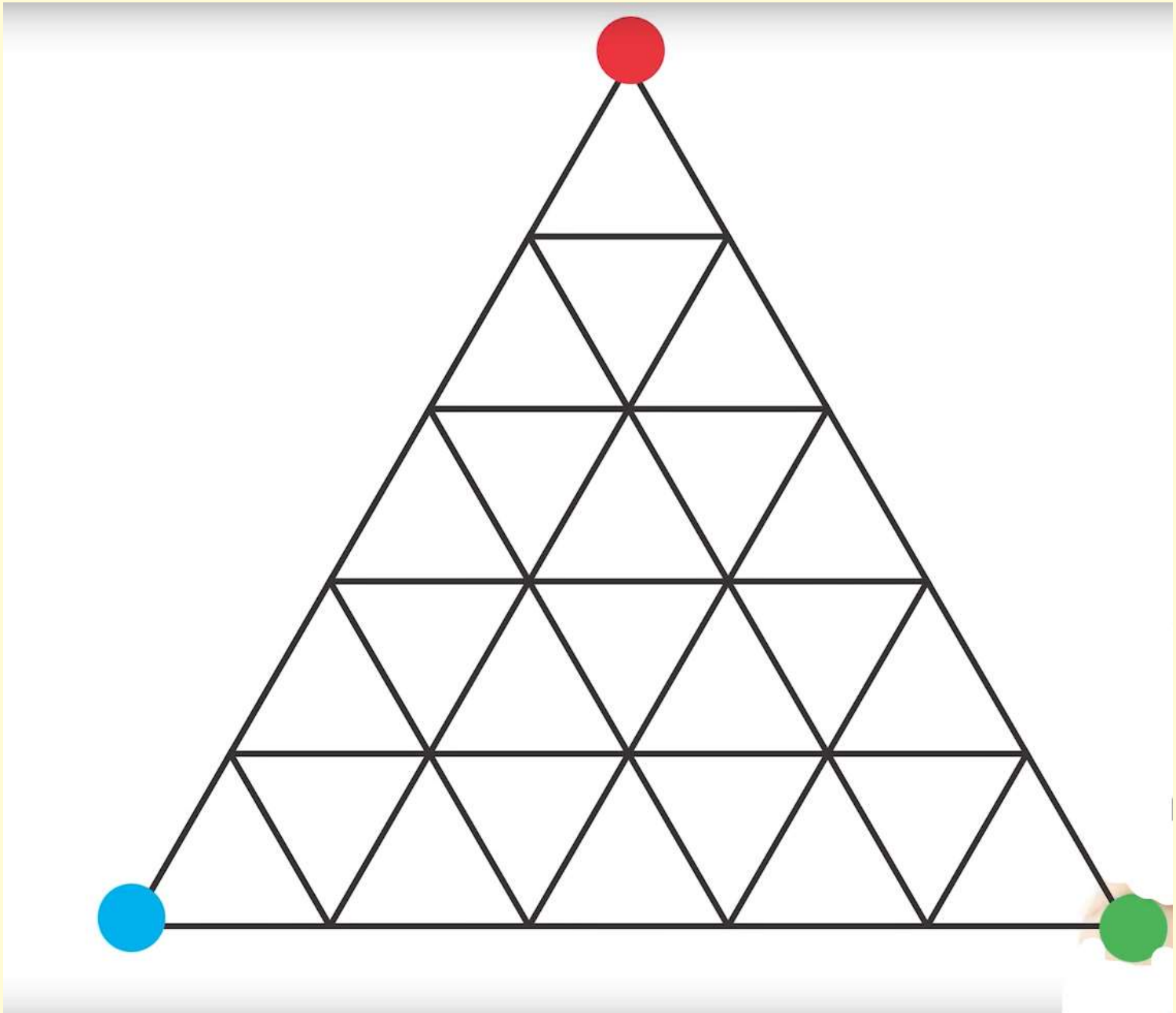




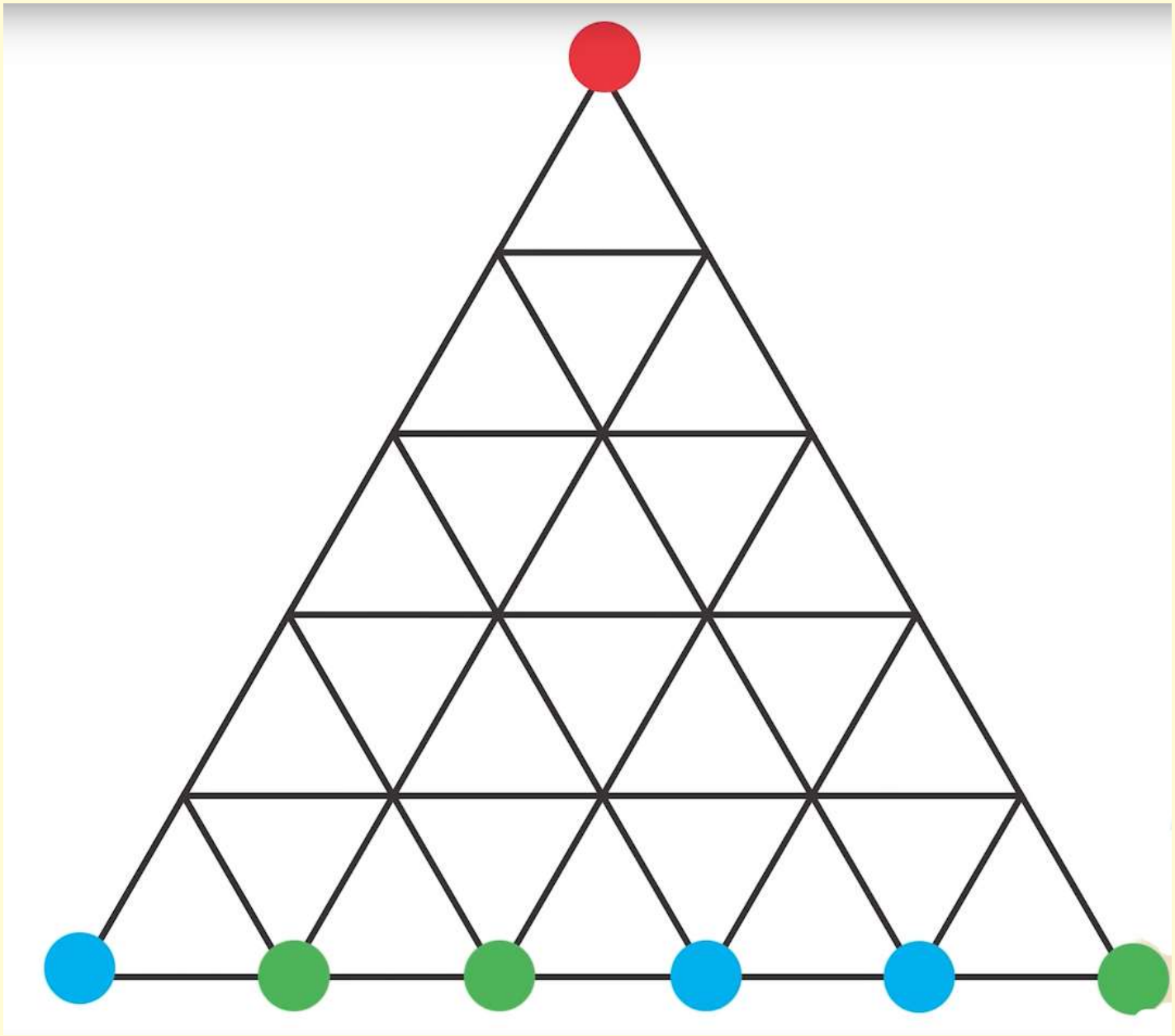
regular triangulation



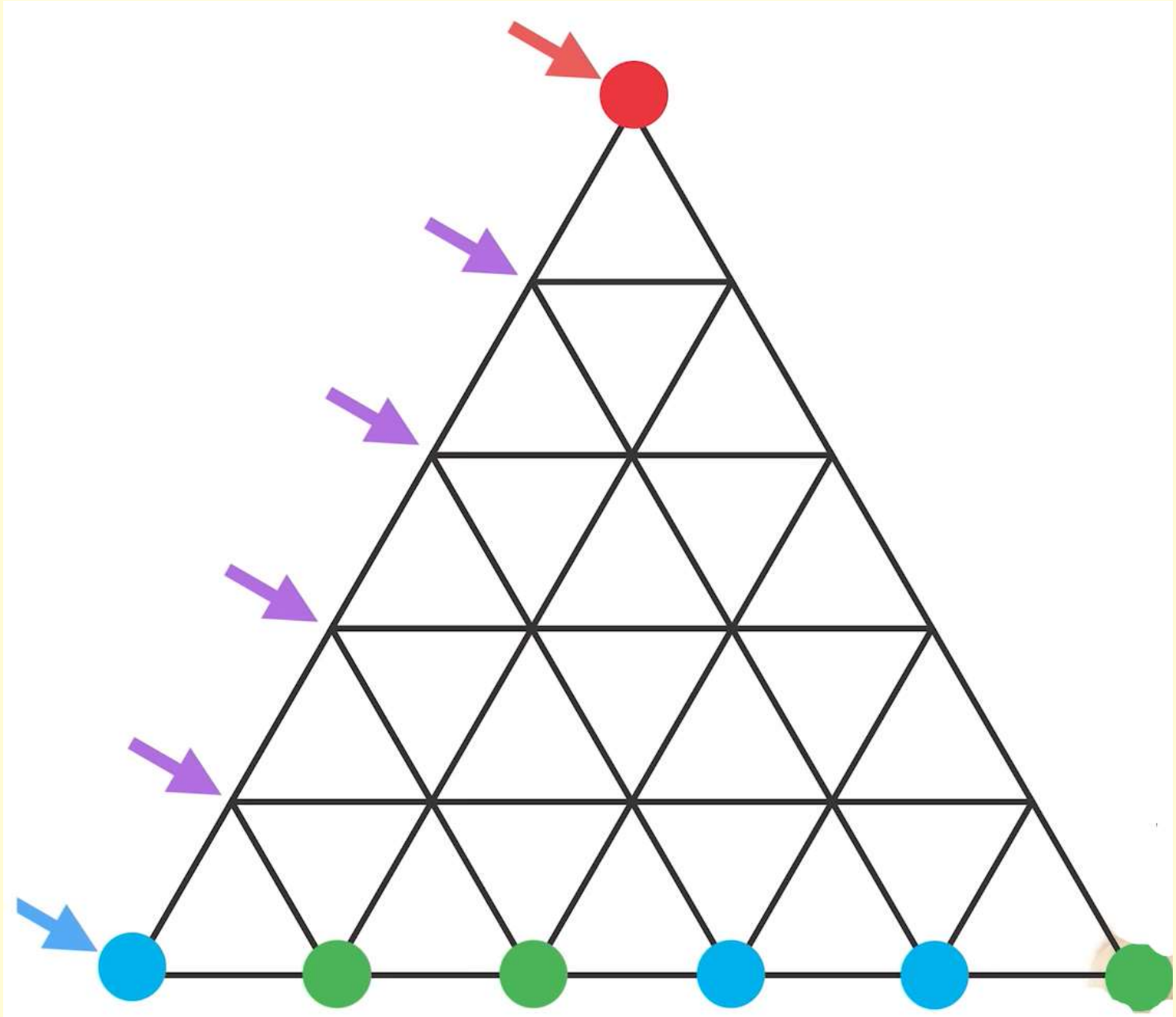
color each vertex of the big Δ differently



color each bottom node blue or green



color each left-edge node blue or red



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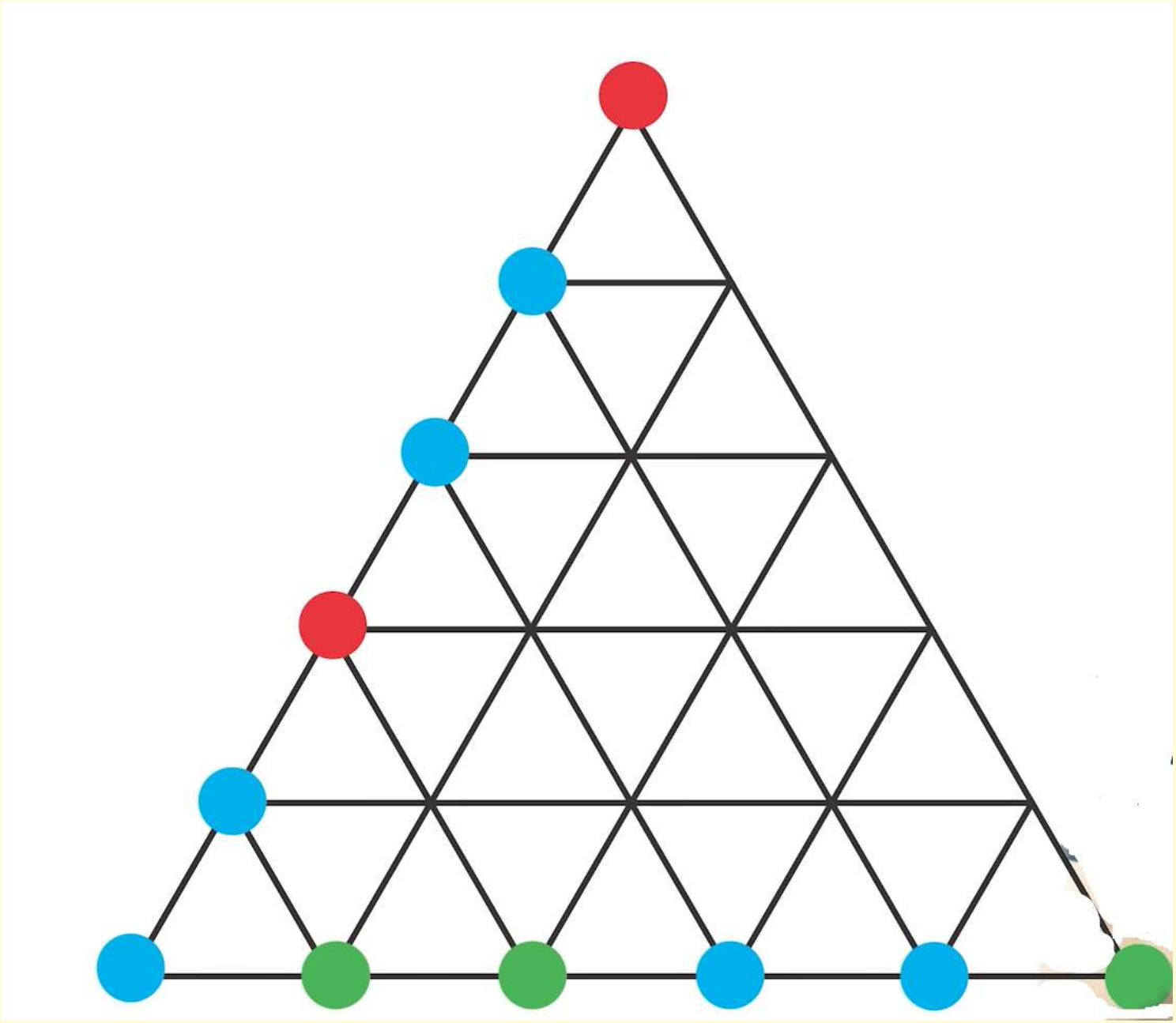
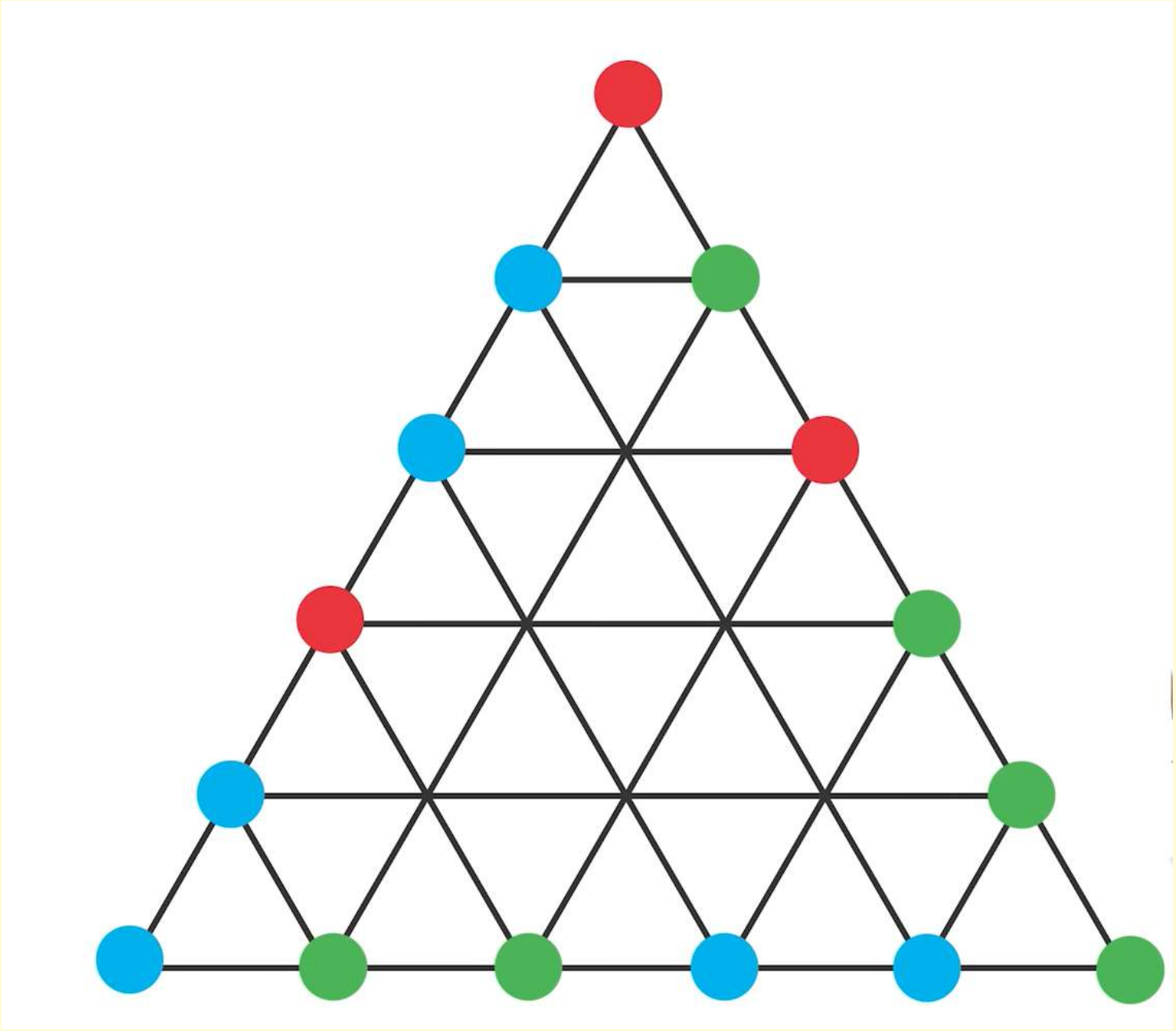
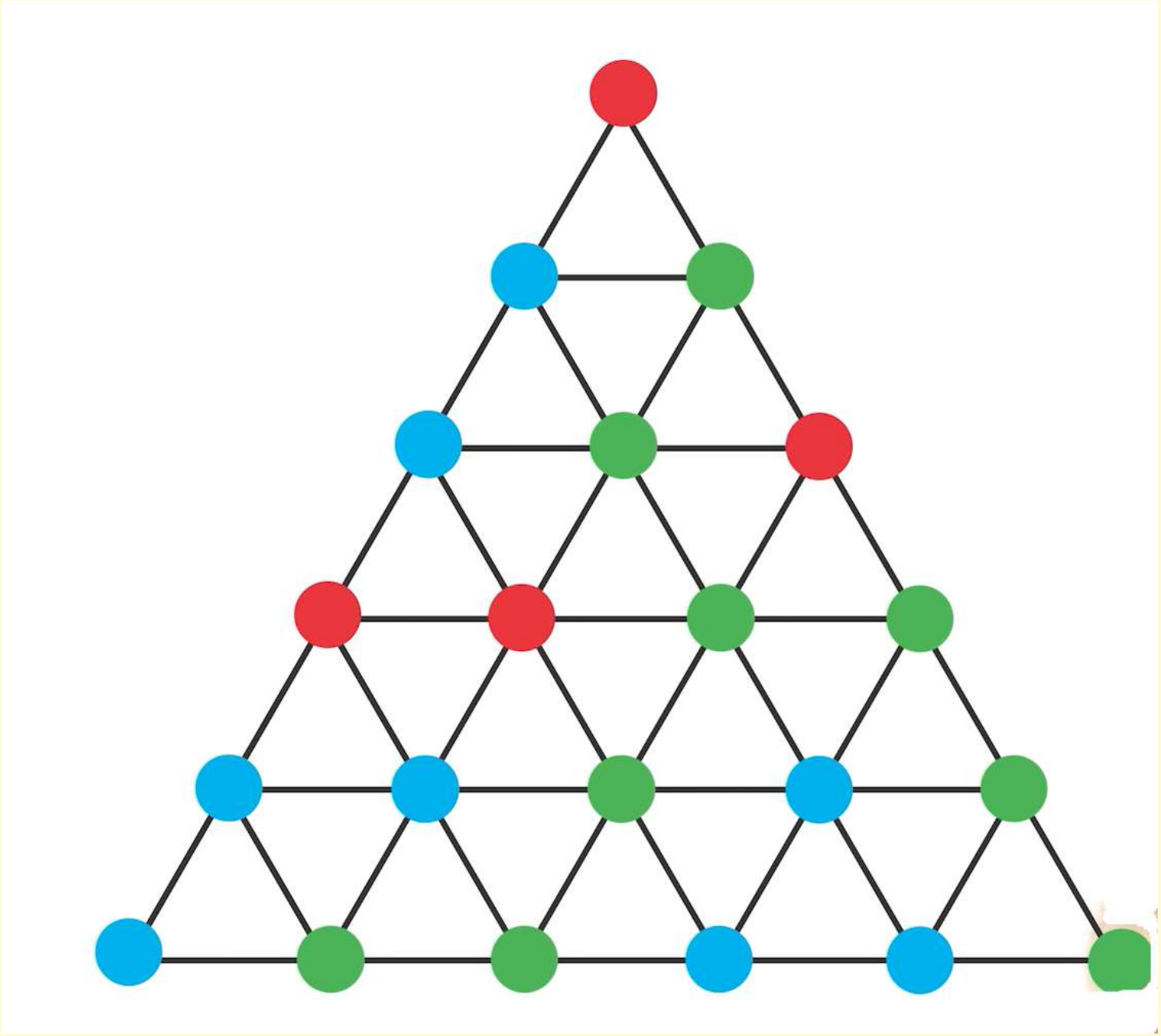


color each right-edge node red or green

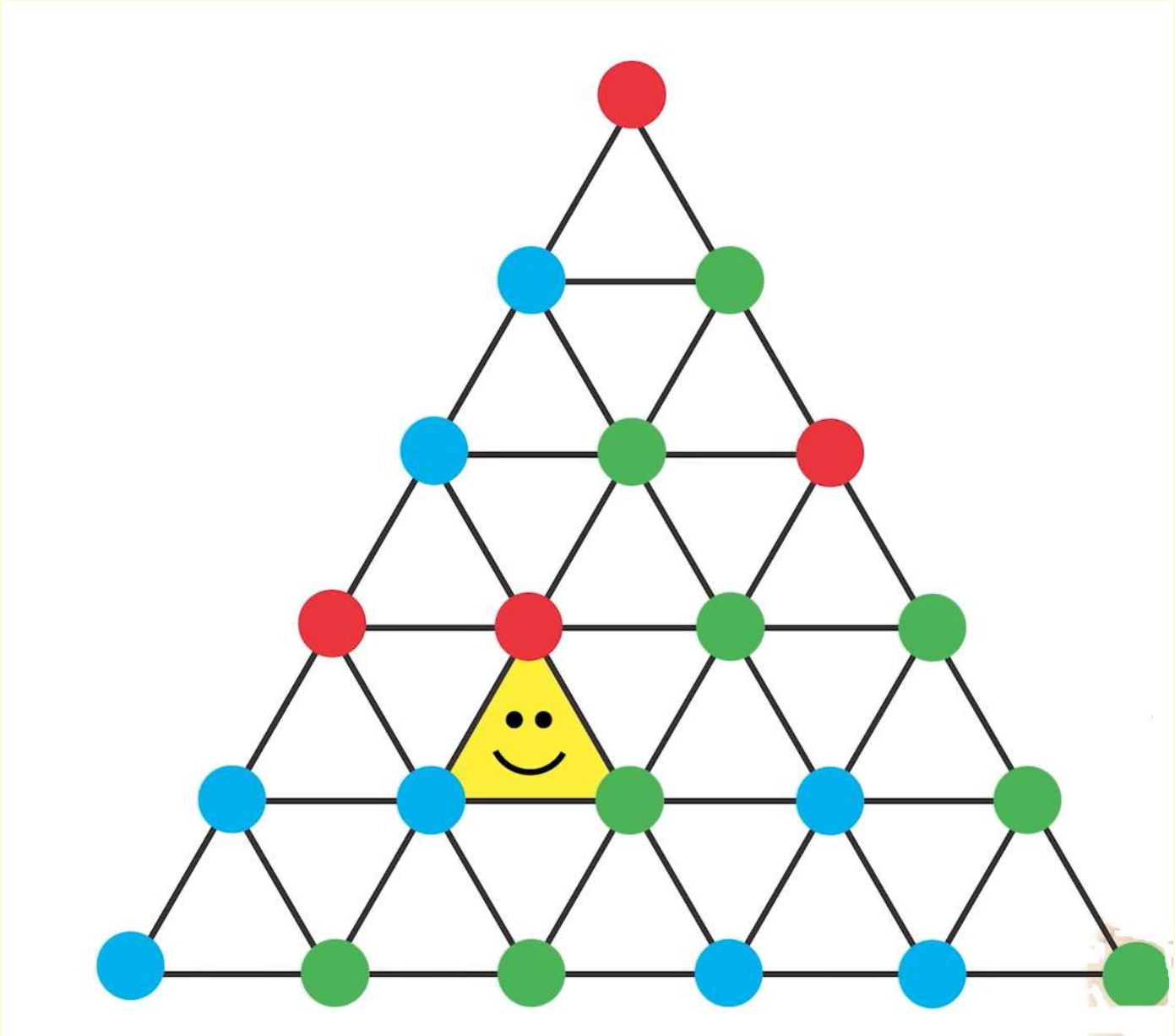


color the interior nodes in any way!

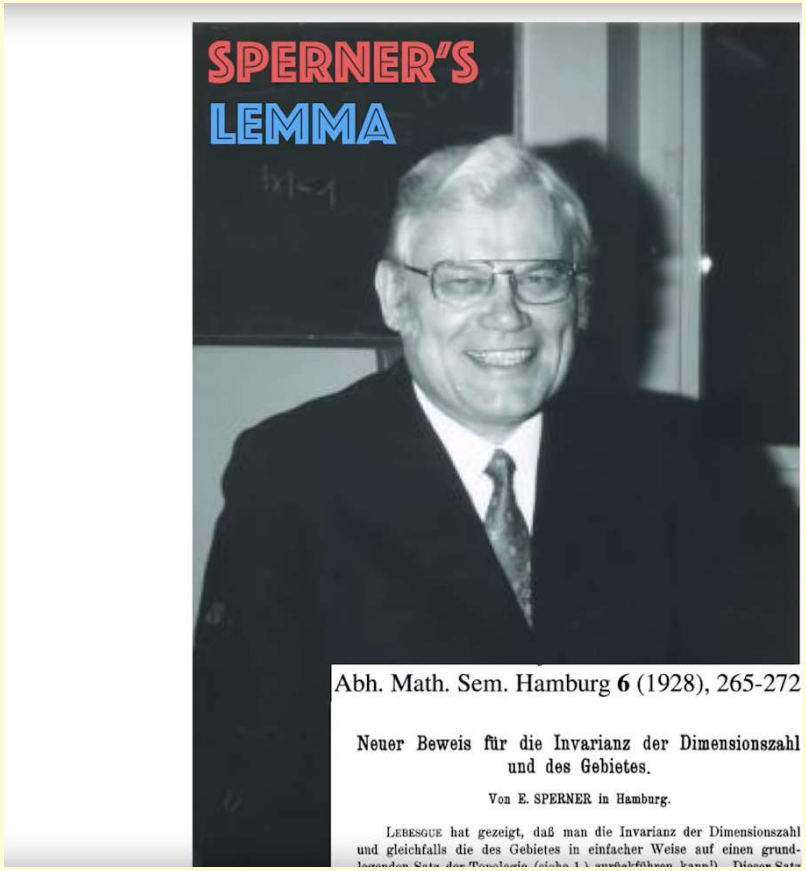


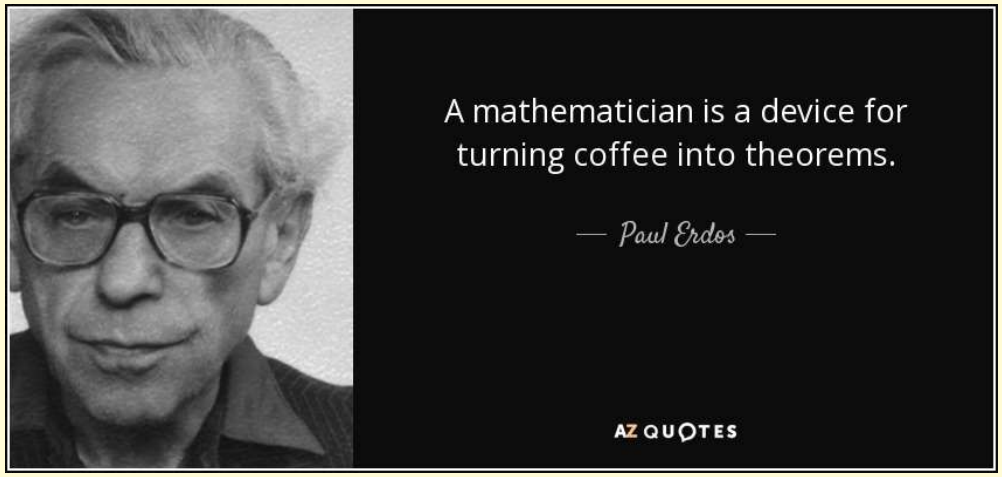
Sperner's Lemma

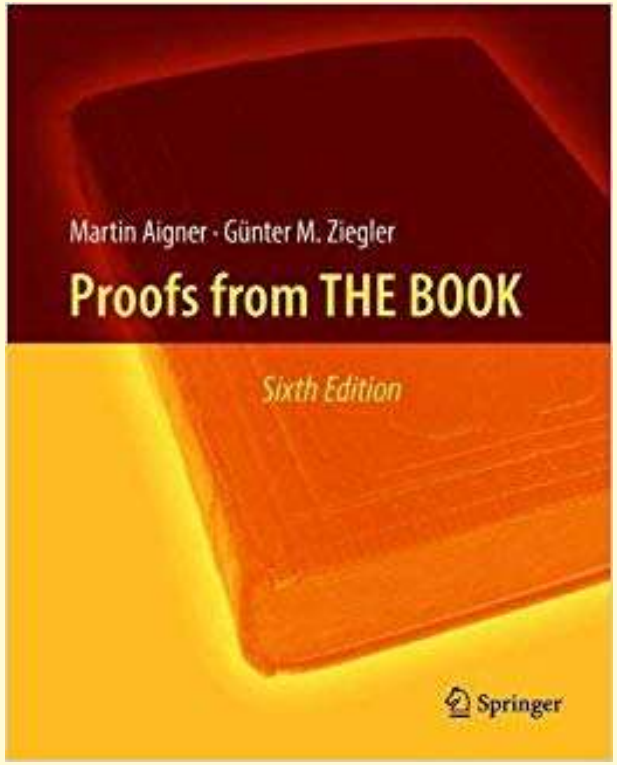
In any triangulation there must be a small "tricolored" triangle



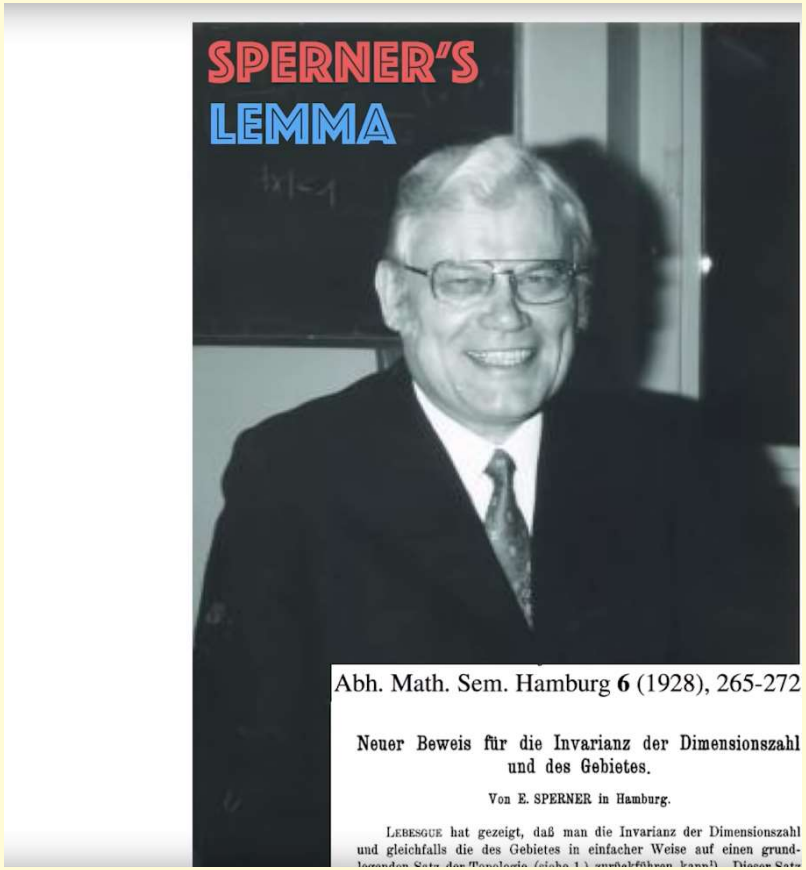
Proof is in The Book







a poetic proof!



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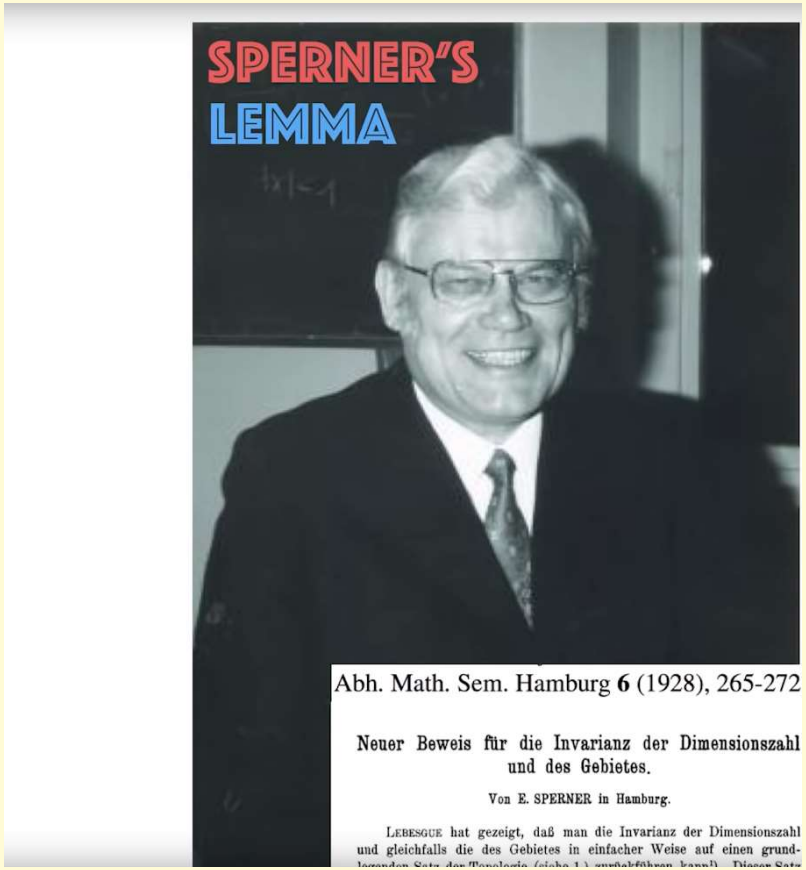


MATEMATİĞİN ŞİİR YÖNÜ

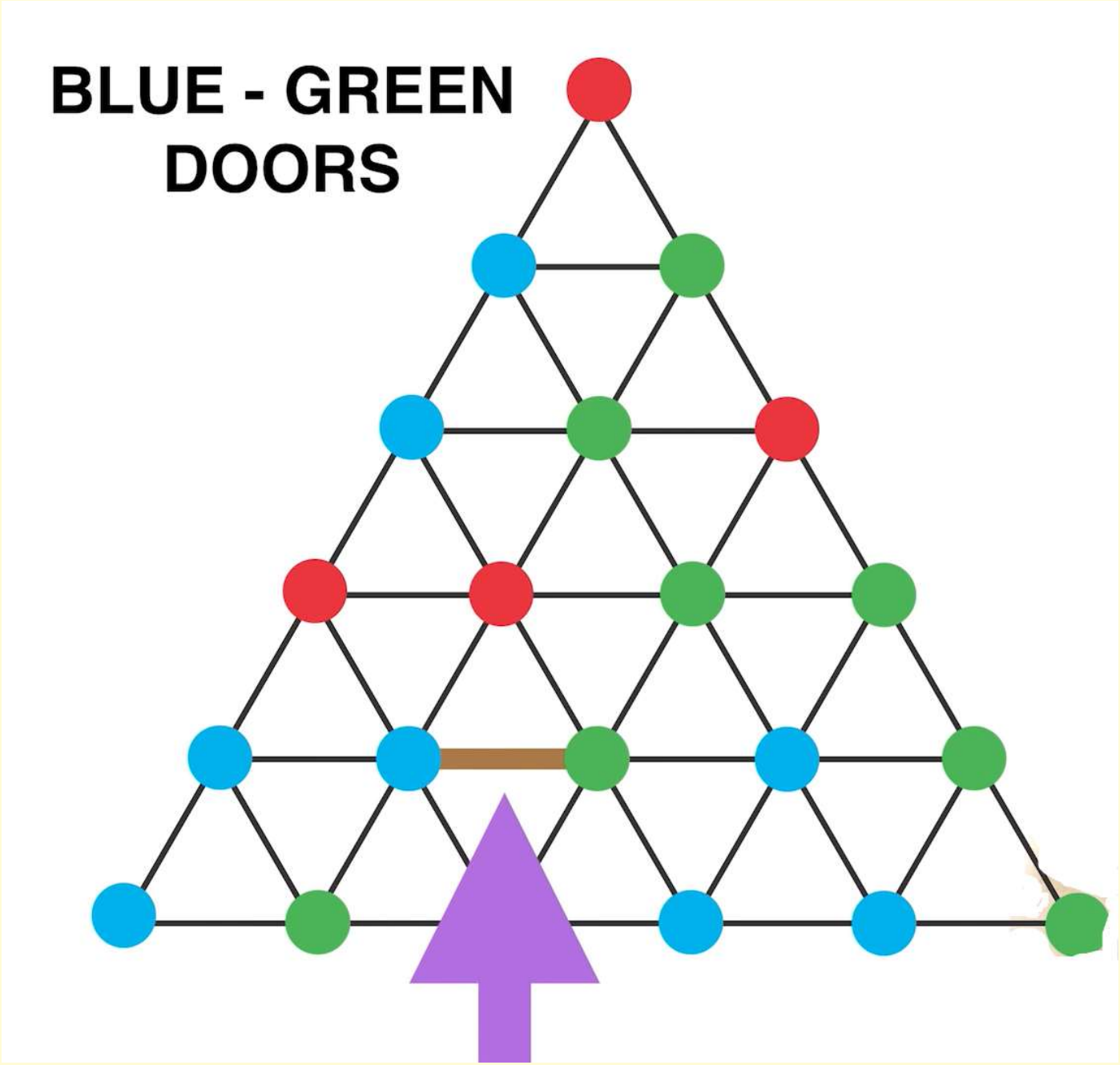
Ord.Prof.Dr. Cahit Arf *
Meydan Dergisi'nin Nisan 1960 sayısında yayınlanan makaleyi aynen sunuyoruz.

"Bir nevi şair olmıyan bir matematikçi, hiç bir zaman mükemmel bir matematikçi olamaz." Weirstrass

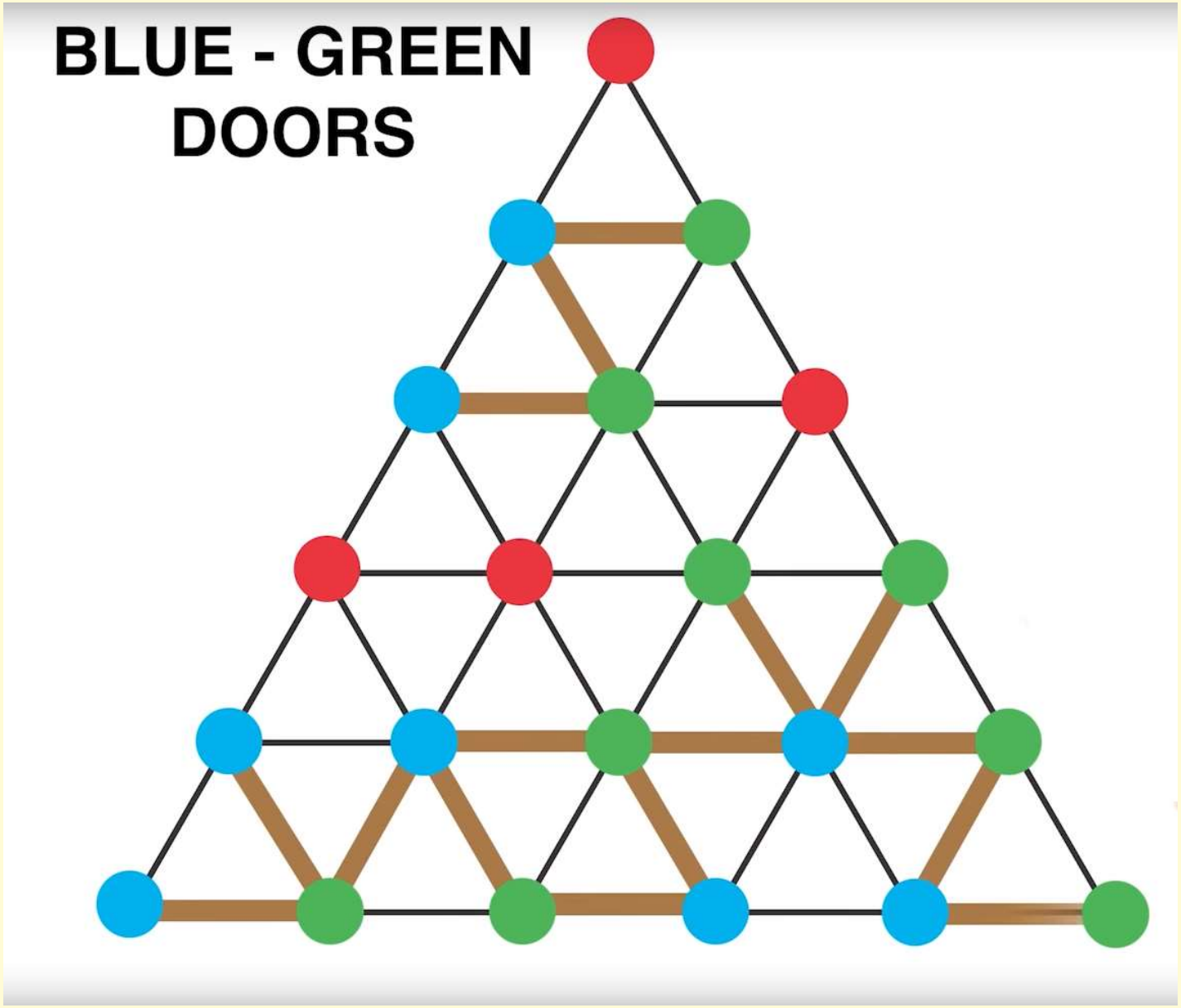
"İyi bakıldığı zaman matematik sadece doğruyu değil (ilâhi) yüksek bir güzelliği de ihtiva eder. Soğuk ve muhteşem bir güzellik, heykellerdeki gibi tabiatımızın zaif taraflarına hitap etmiyen, resmin veya müziğin süslü taraflarına malik olmıyan, fakat süblim bir safiyeti (temizliği) olan en büyük sanat'ın erişebileceği en büyük mükemmeliyete erişebilen bir güzellik." Bertrand Russell

bu kitapların zaruri olarak kullanıldıkları stil matematik hakkında yanlış bir fikir verebilir ve matematiğin bir bilmece kolleksiyonu olduğu zehabını uyandırabilir. Halbuki matematiğin bunun tam zıddı olduğunu, yani insanın karşılaştığı muammalardan kurtulmak daha doğrusu bunları muamma olmaktan çıkarmak için sarf ettiği gayretlerin mahsulü olan bir zihin terbiyesi olduğunu söyliyebiliriz. Meselâ n tam sayısı 2 den büyük olmak üzere $x^n + y^n = z^n$ olacak şekilde x, y, z gibi üç tam sayısının mevcut olmadığını ifade eden meşhur Fermat hipotezini ispata teşebbüs eden binlerce amatörün mevcudiyetine mukabil en büyük matematik dehası olarak kabul edilen Gaus'un bu

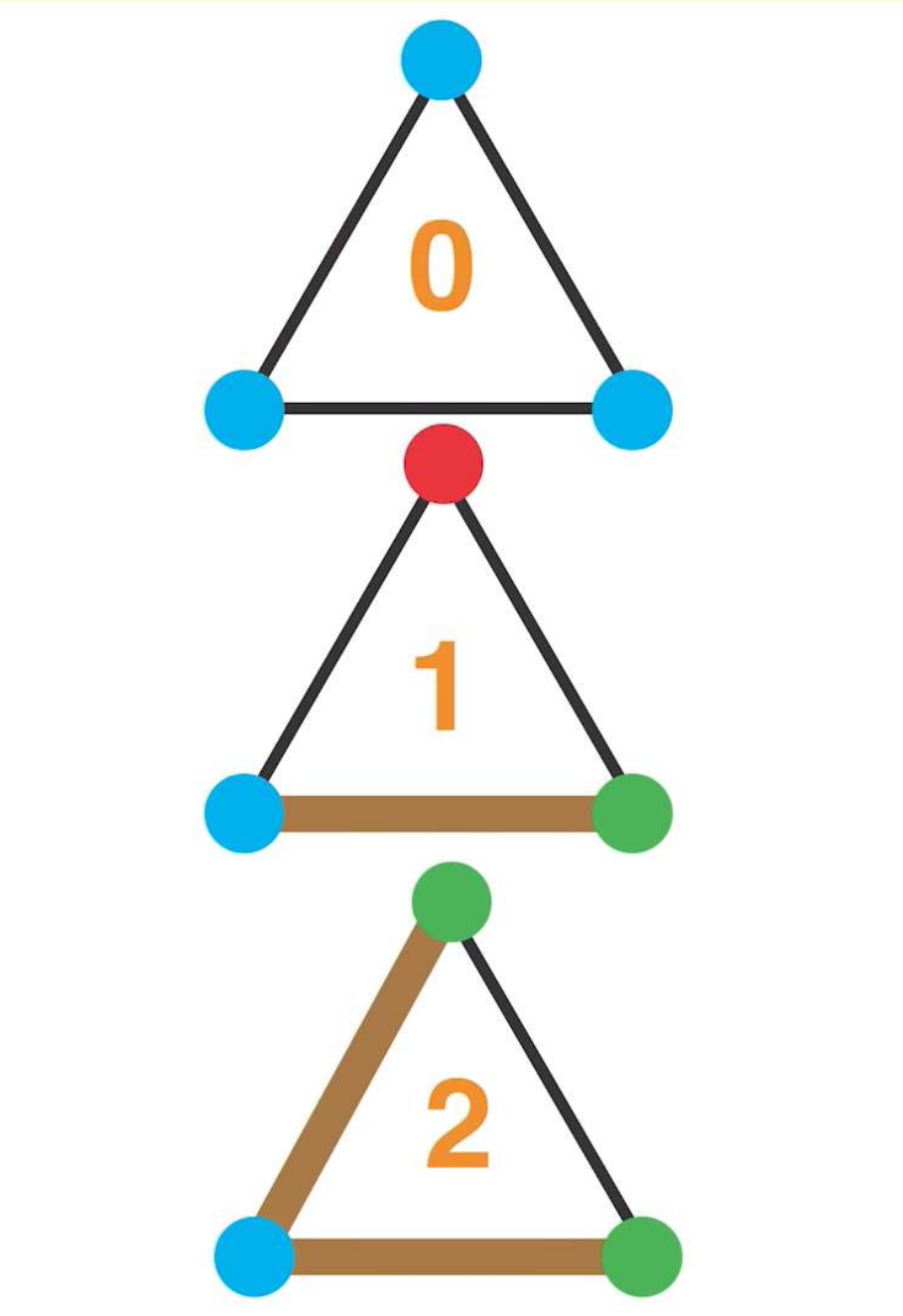
Proof



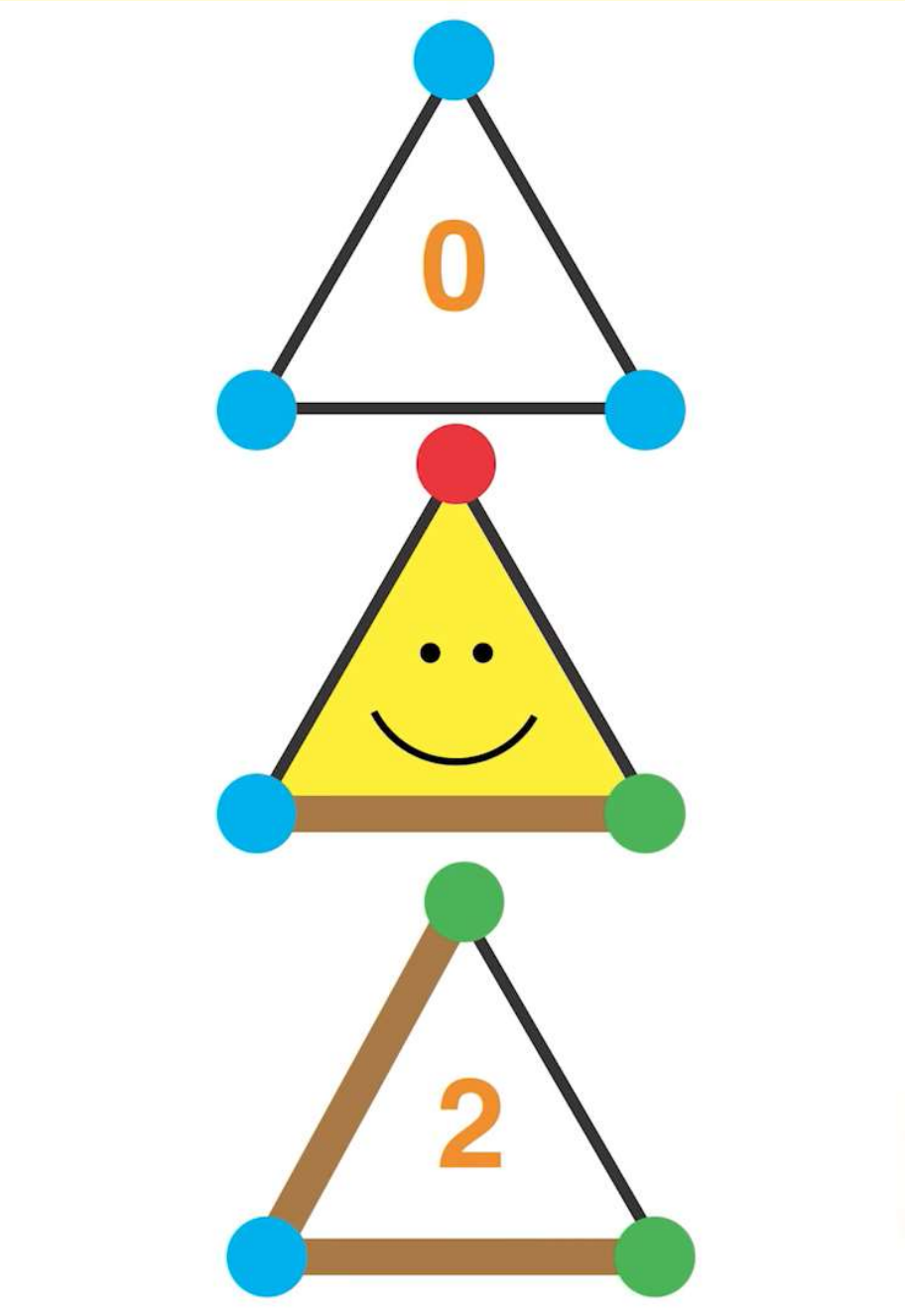
all the doors



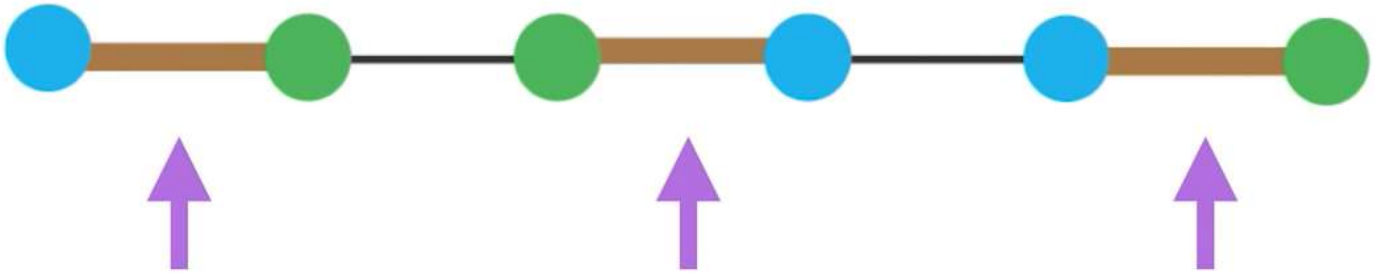
3 possibilities



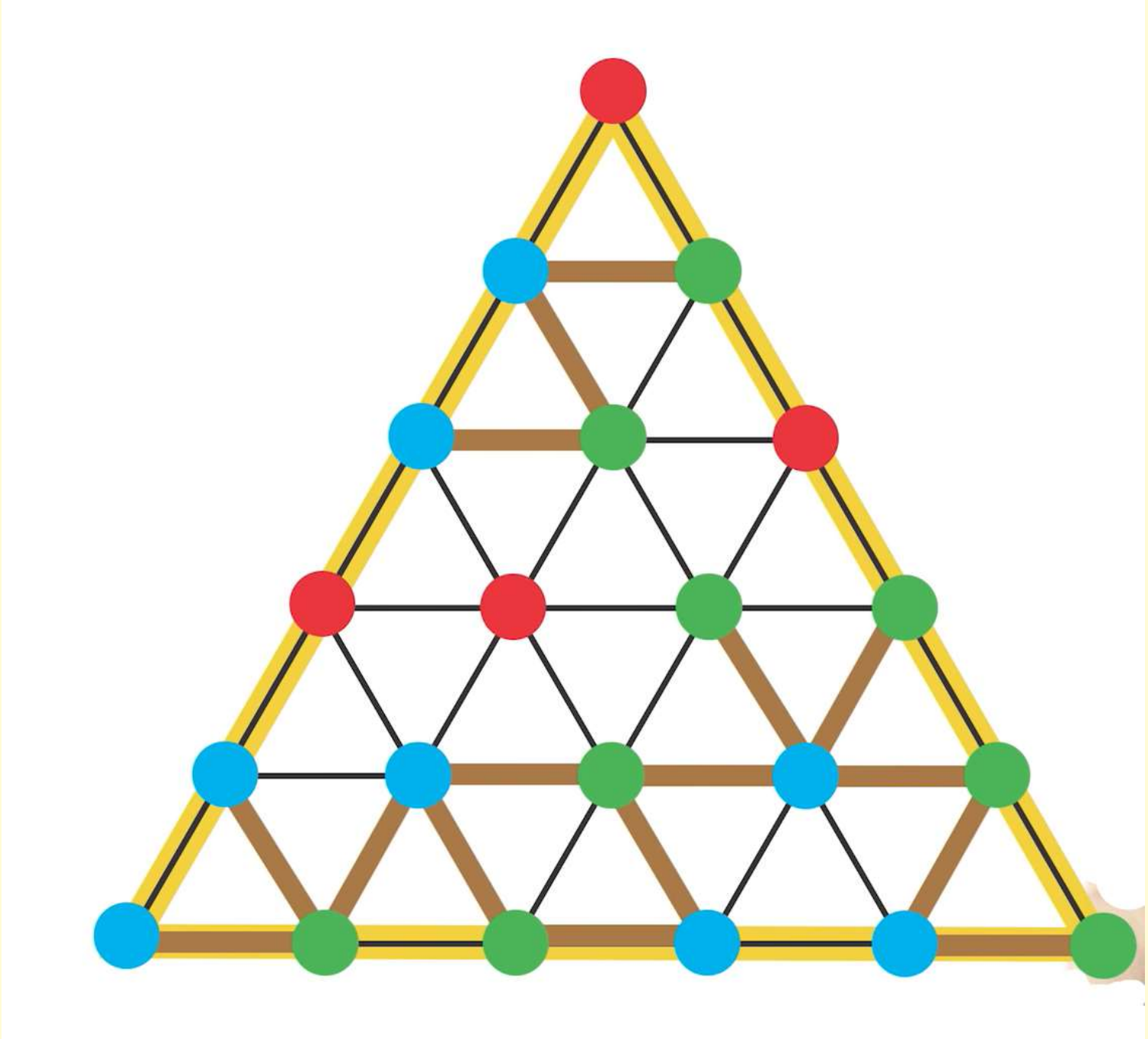
1 door= tricolored triangle



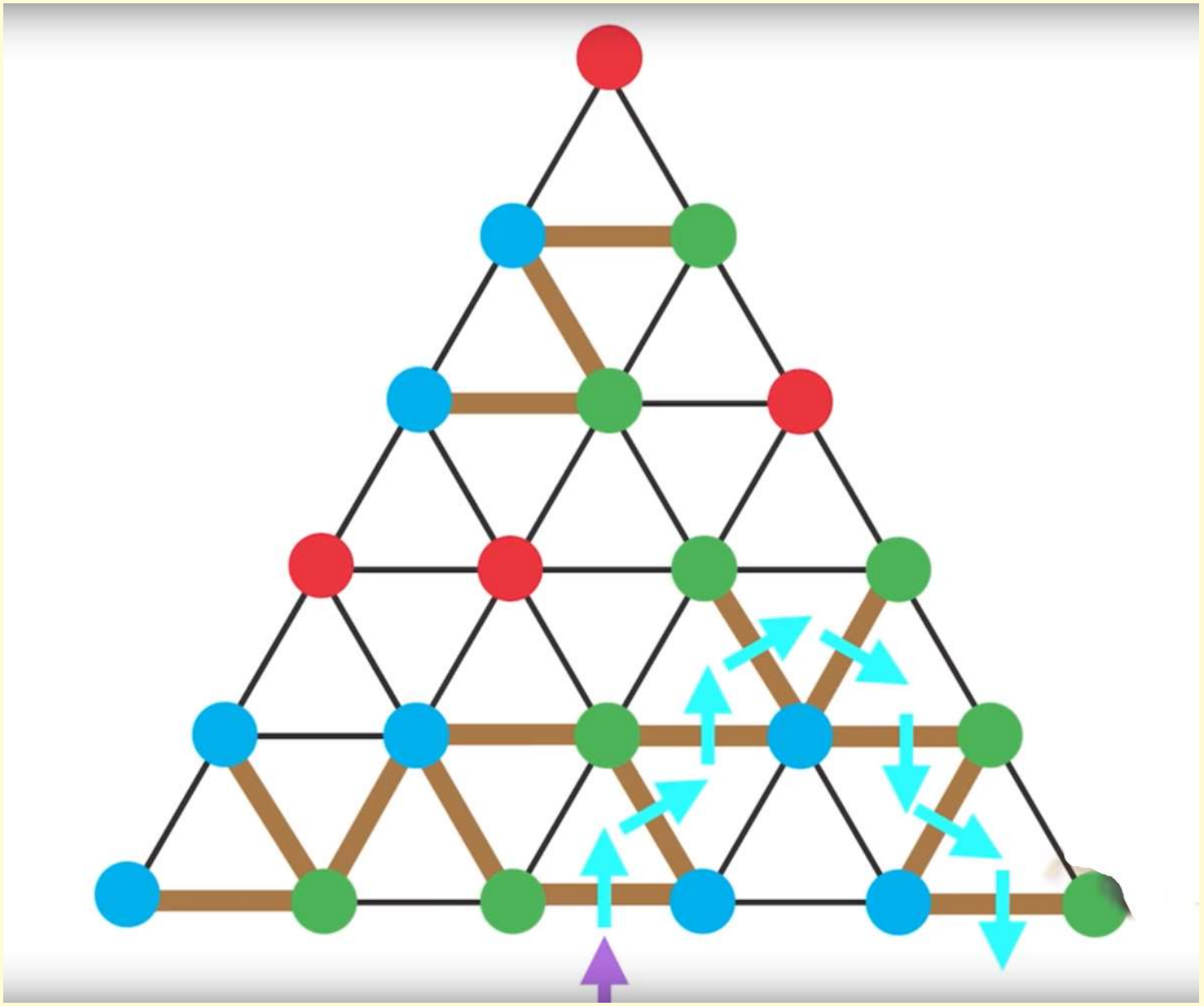
The number of main doors must be odd



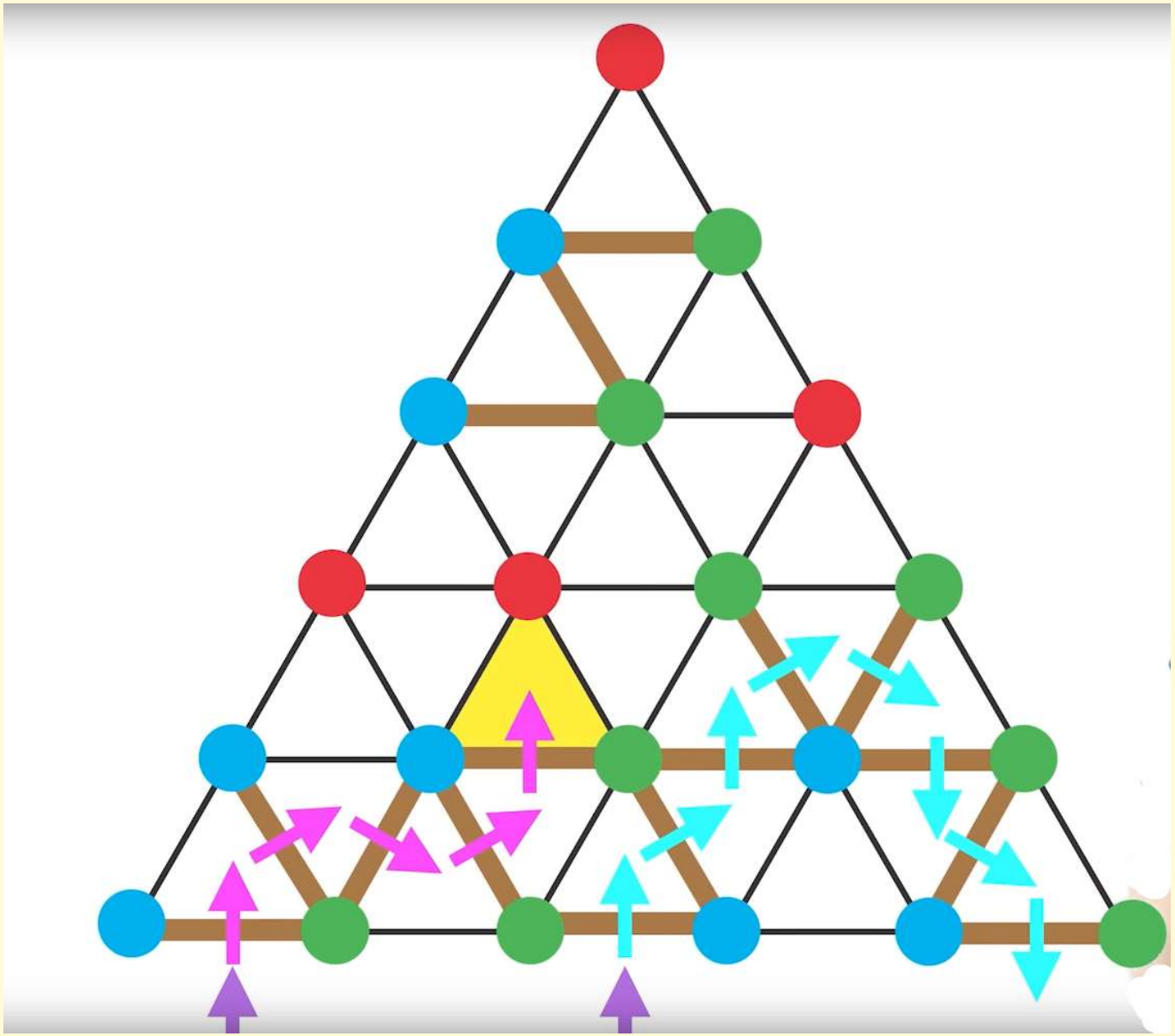
the house, rooms, and the doors



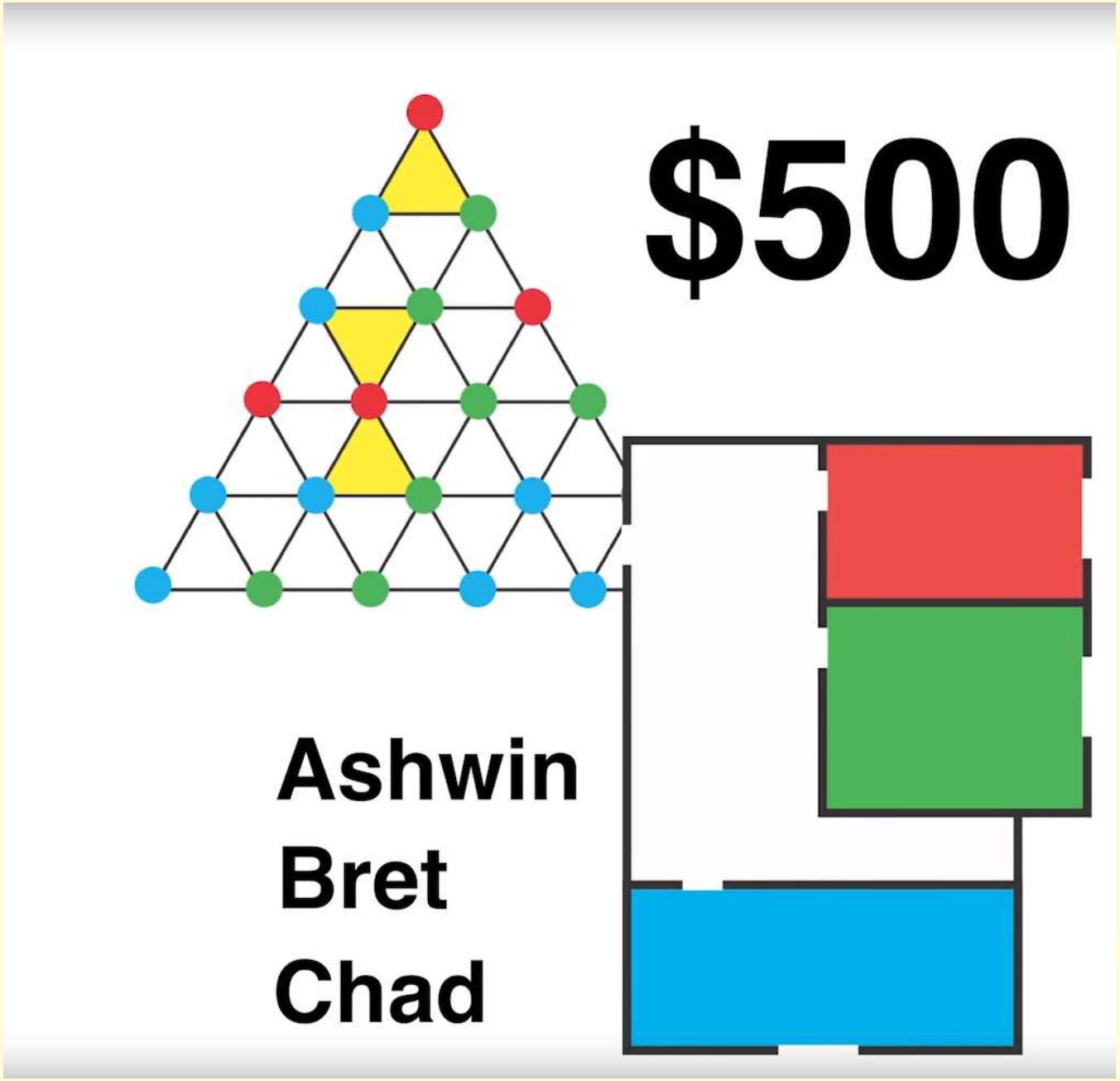
walk through the rooms



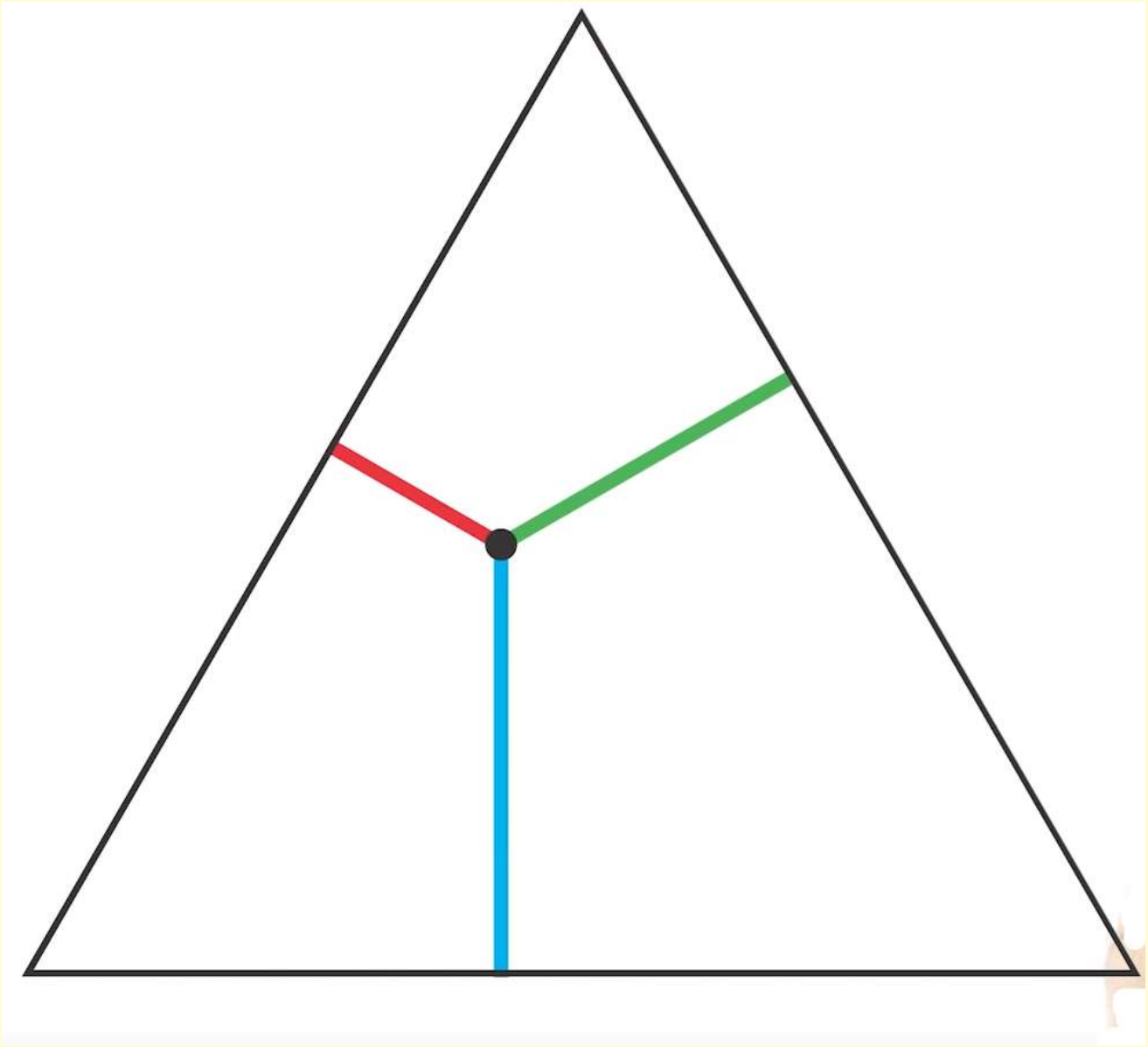
QED



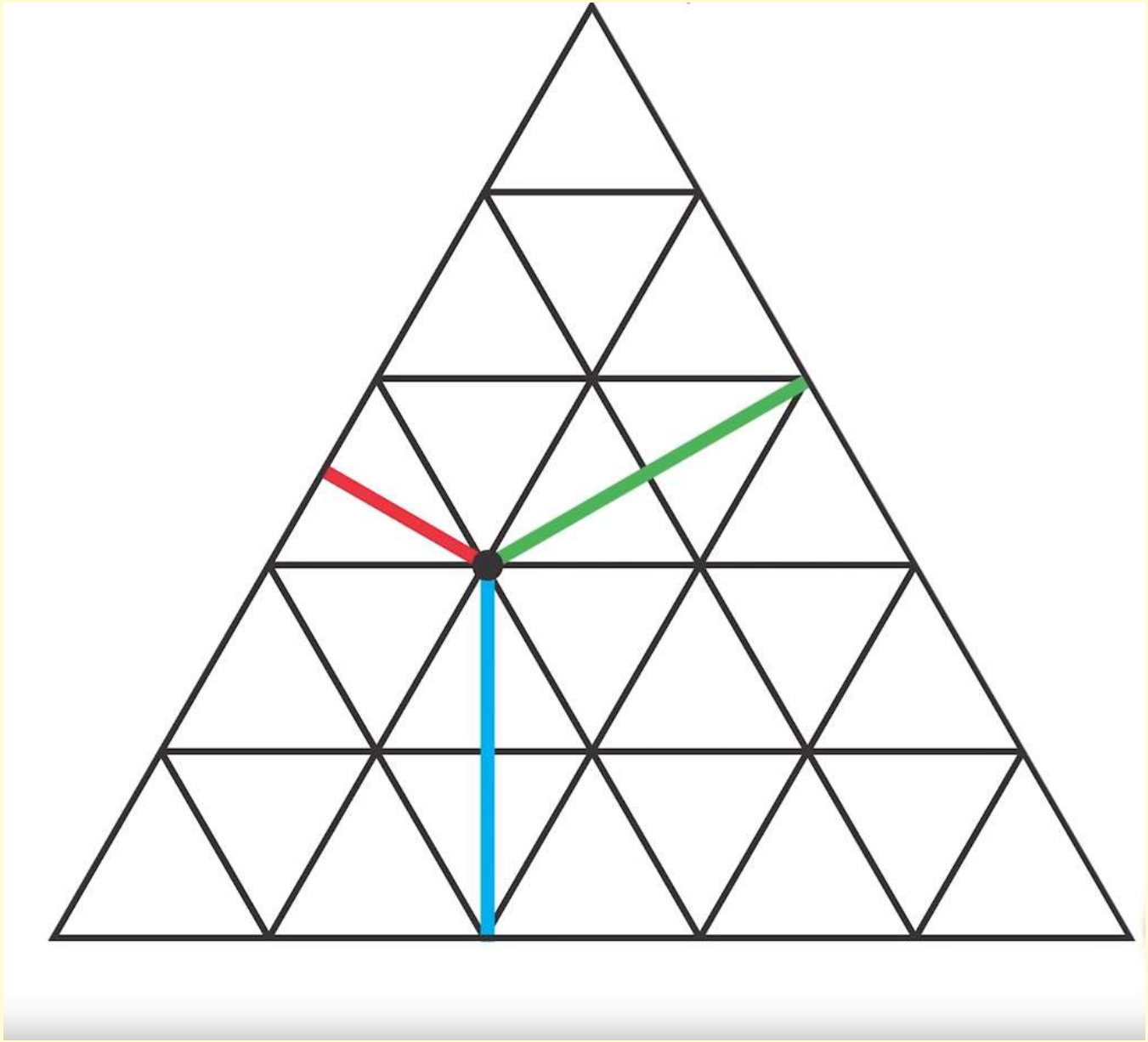
Connection to rental harmony?



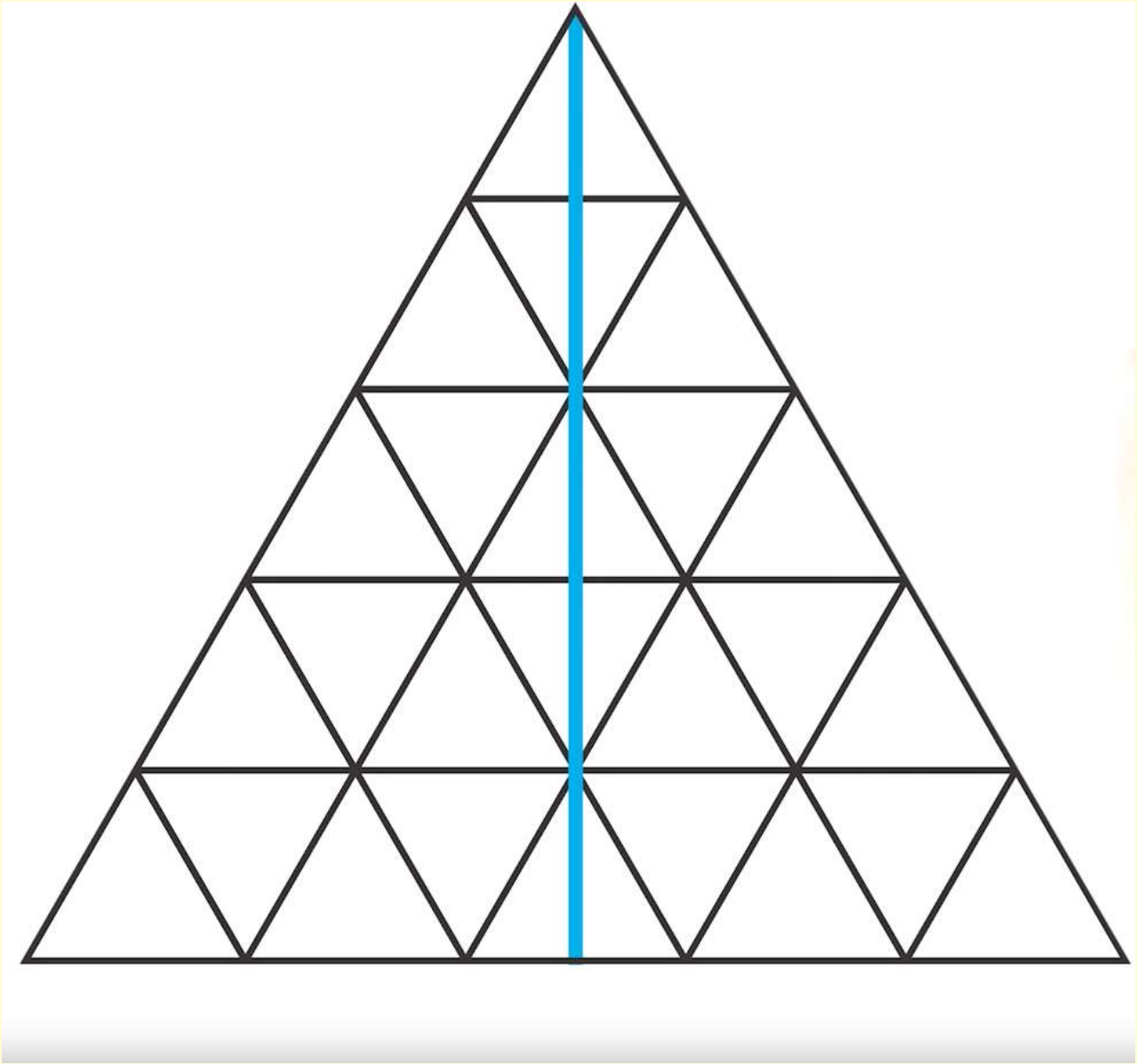
Viviani's Thm: R+B+G=Constant



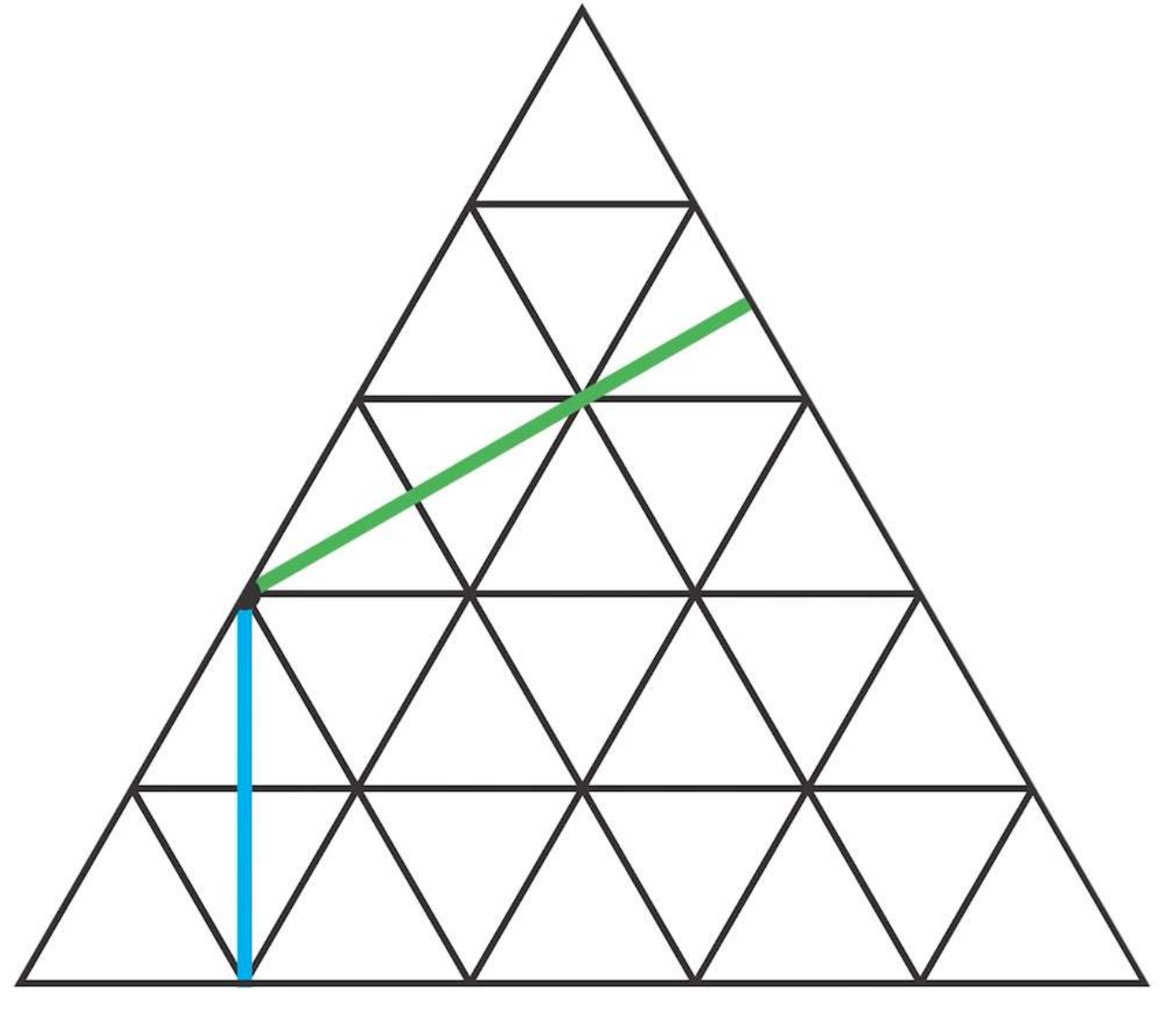
$1+2+2=5$



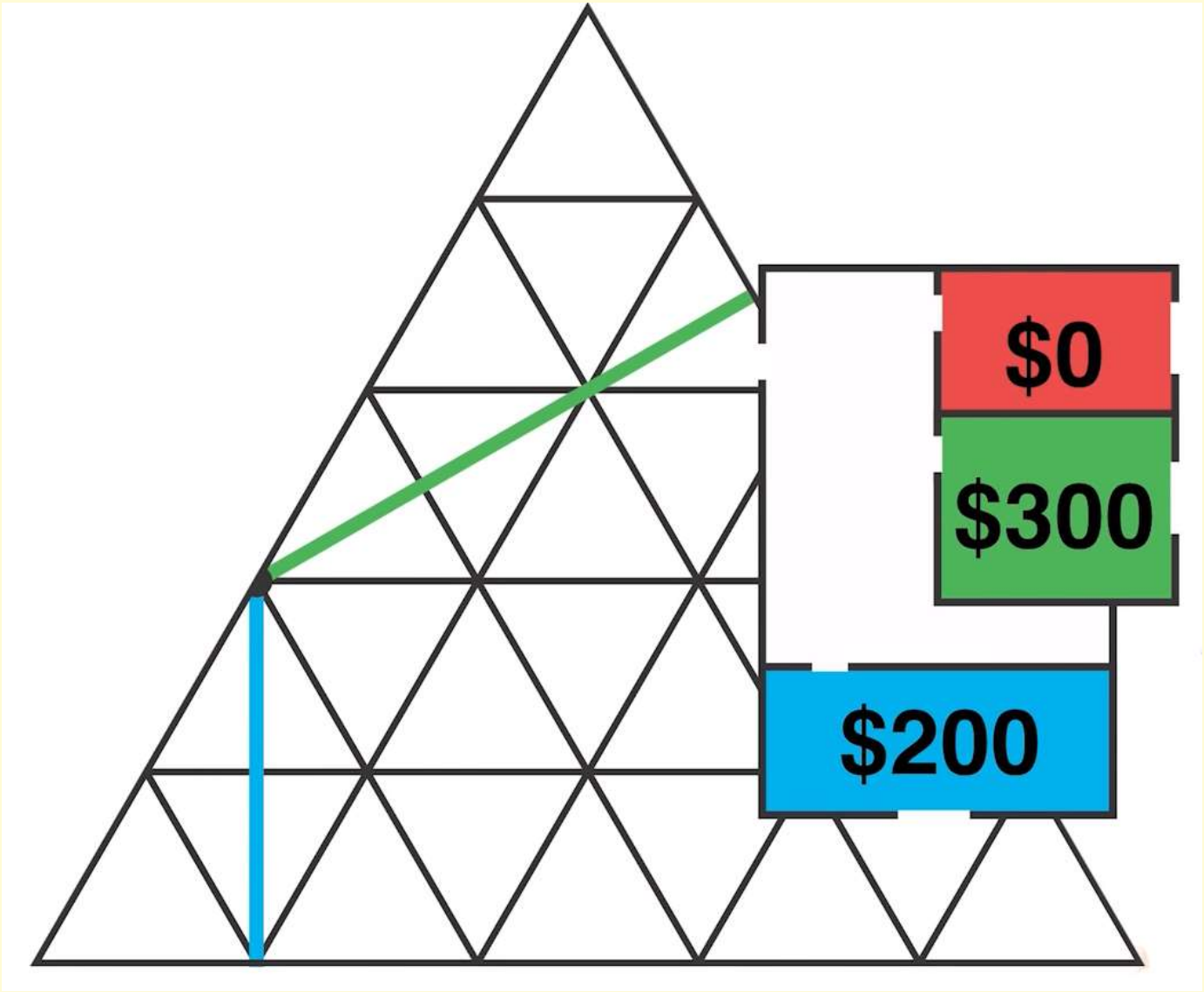
$0+5+0=5$



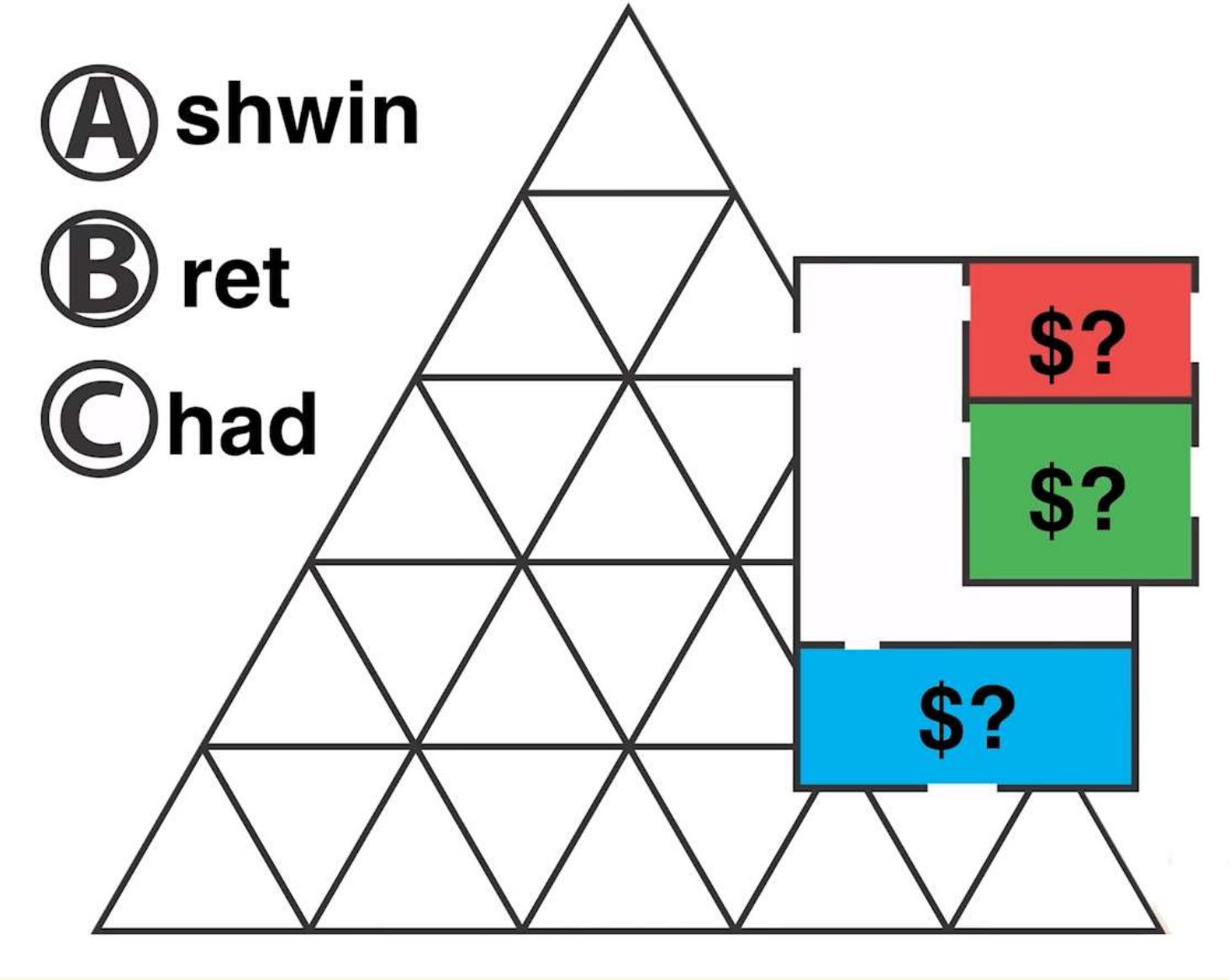
$0+2+3=5$

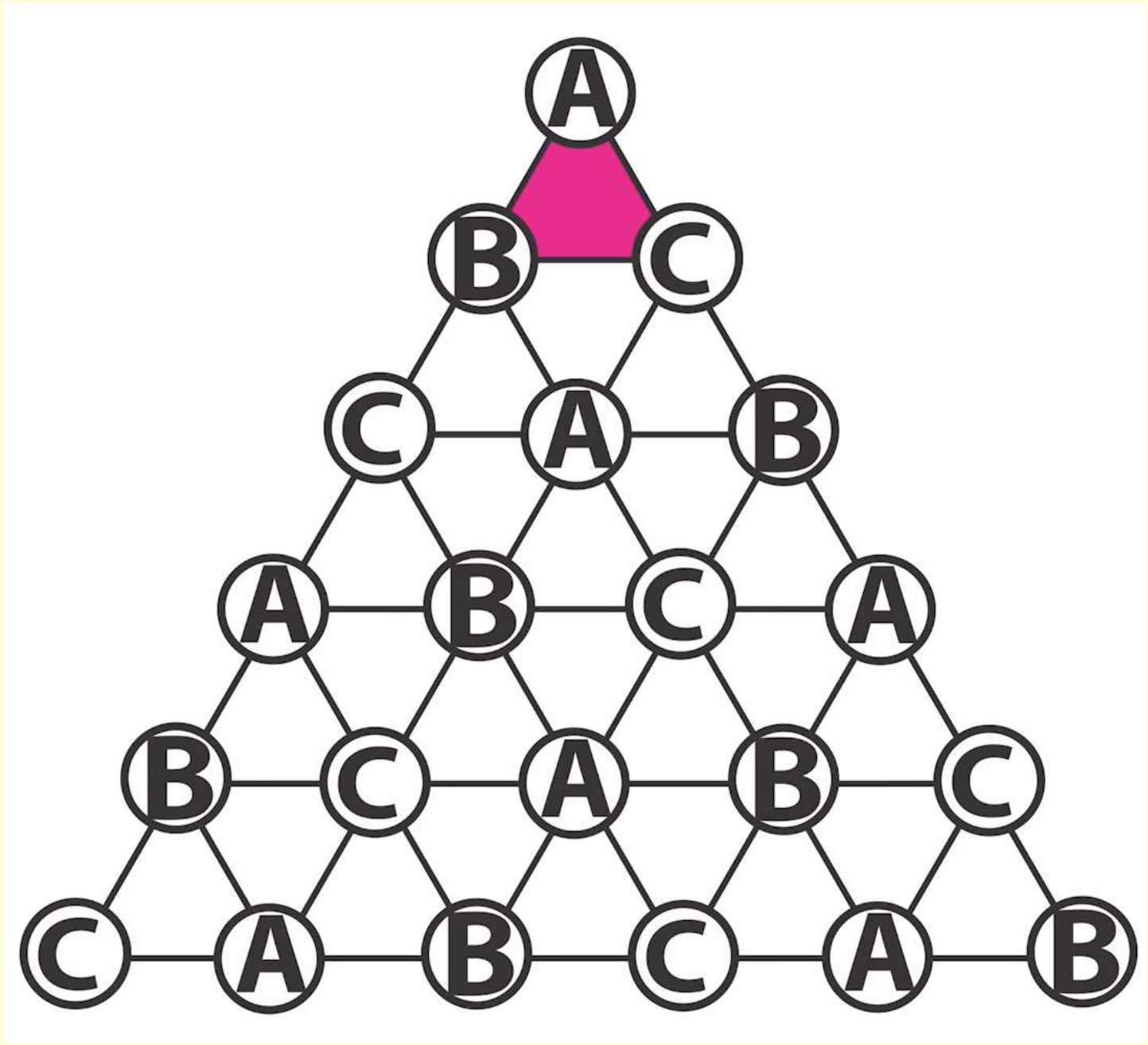


$0+200+300=500$



Rent allocation: ?+?+?=500




A
B C
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B C A B C
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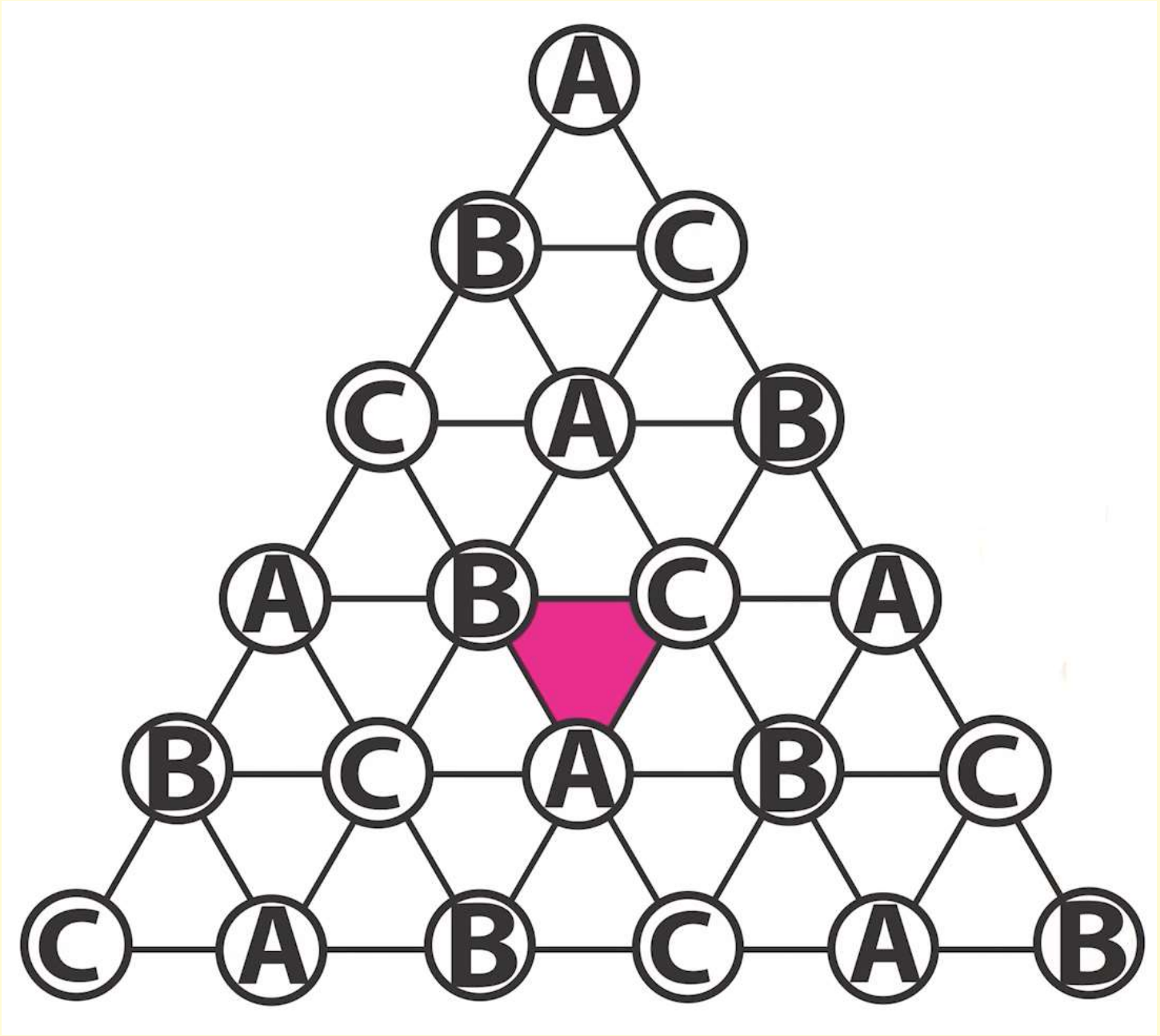
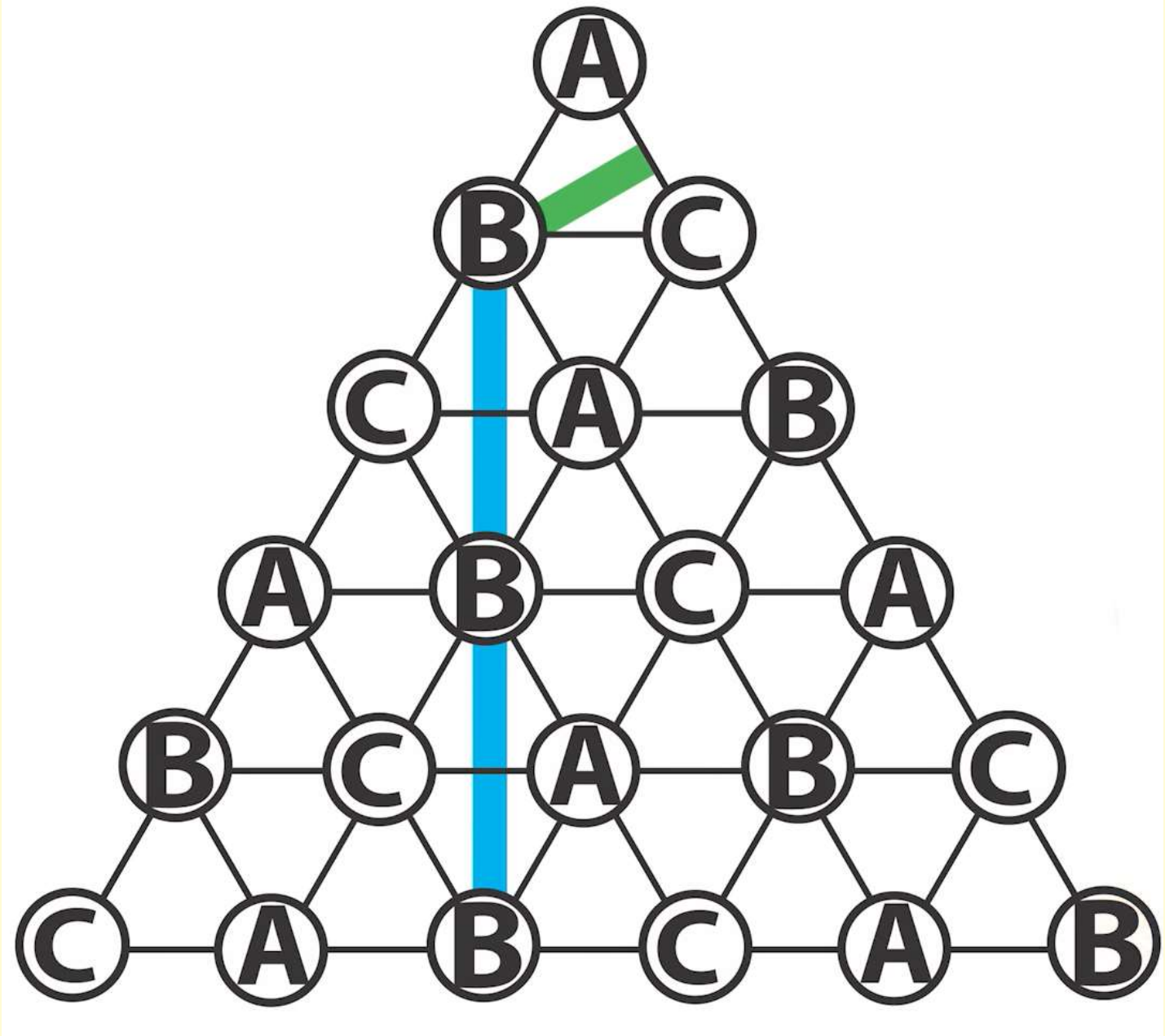
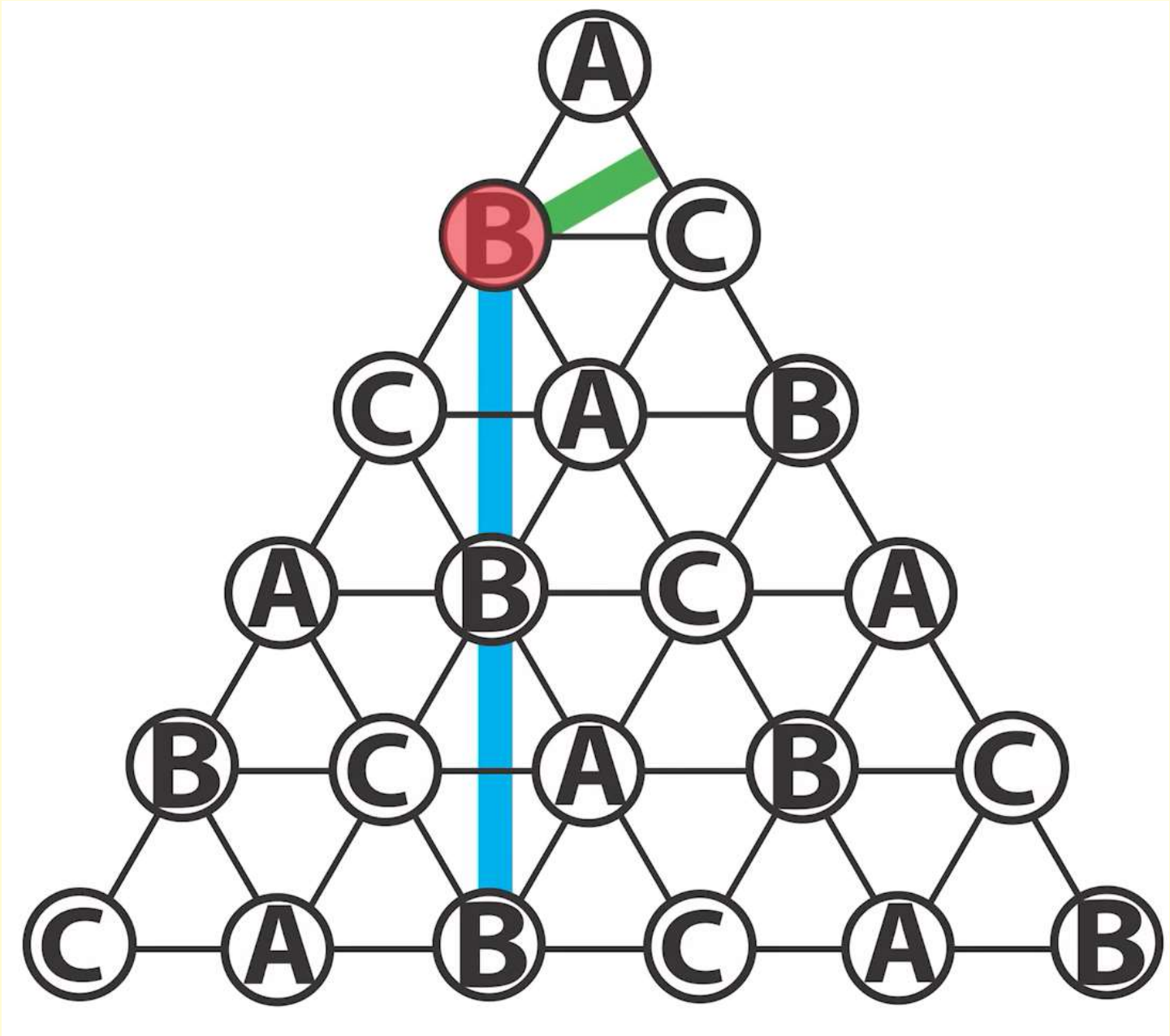

A
B C
C A B
A B C A
B C A B C
C A B C A B

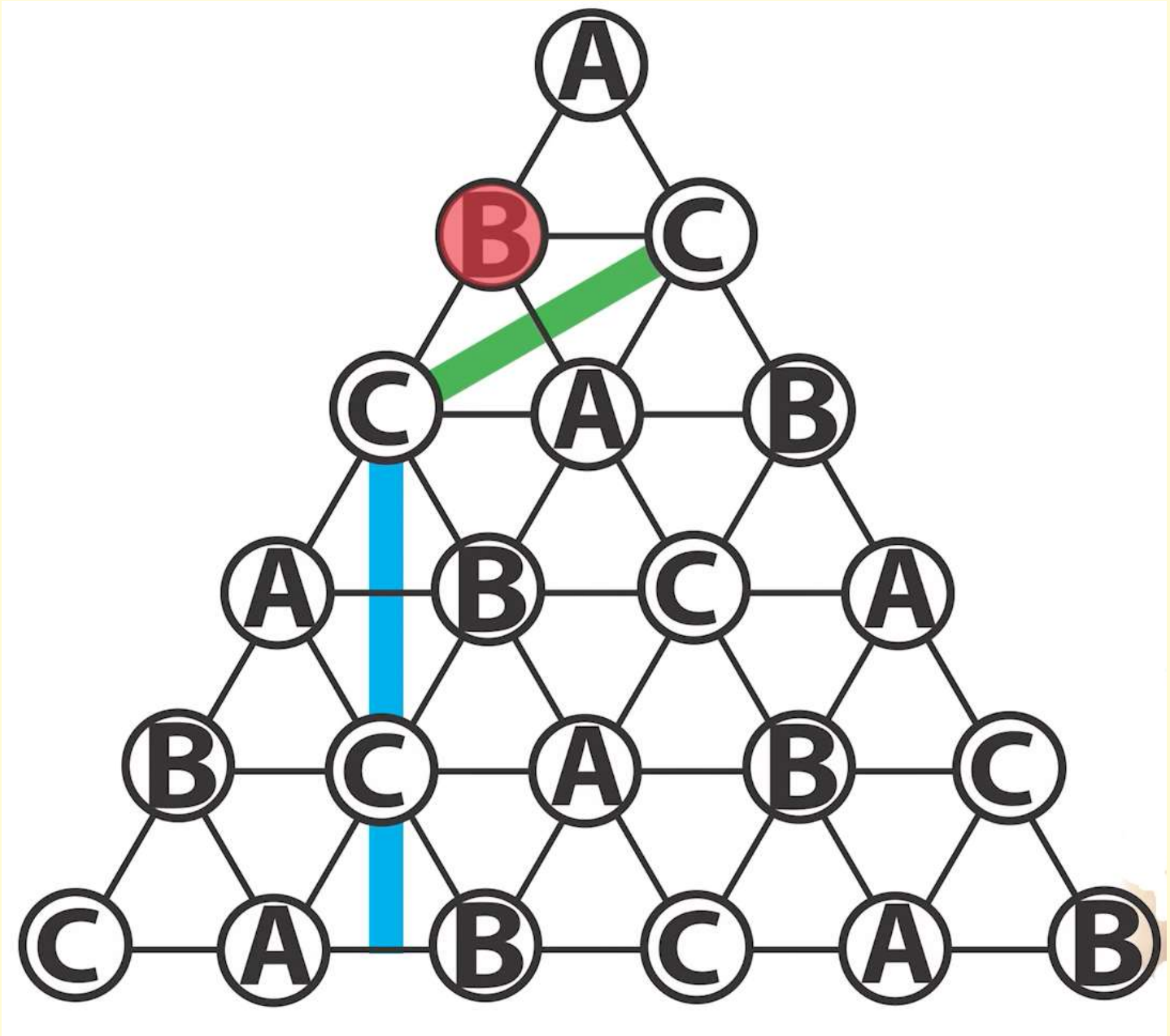
$0+400+100=500$



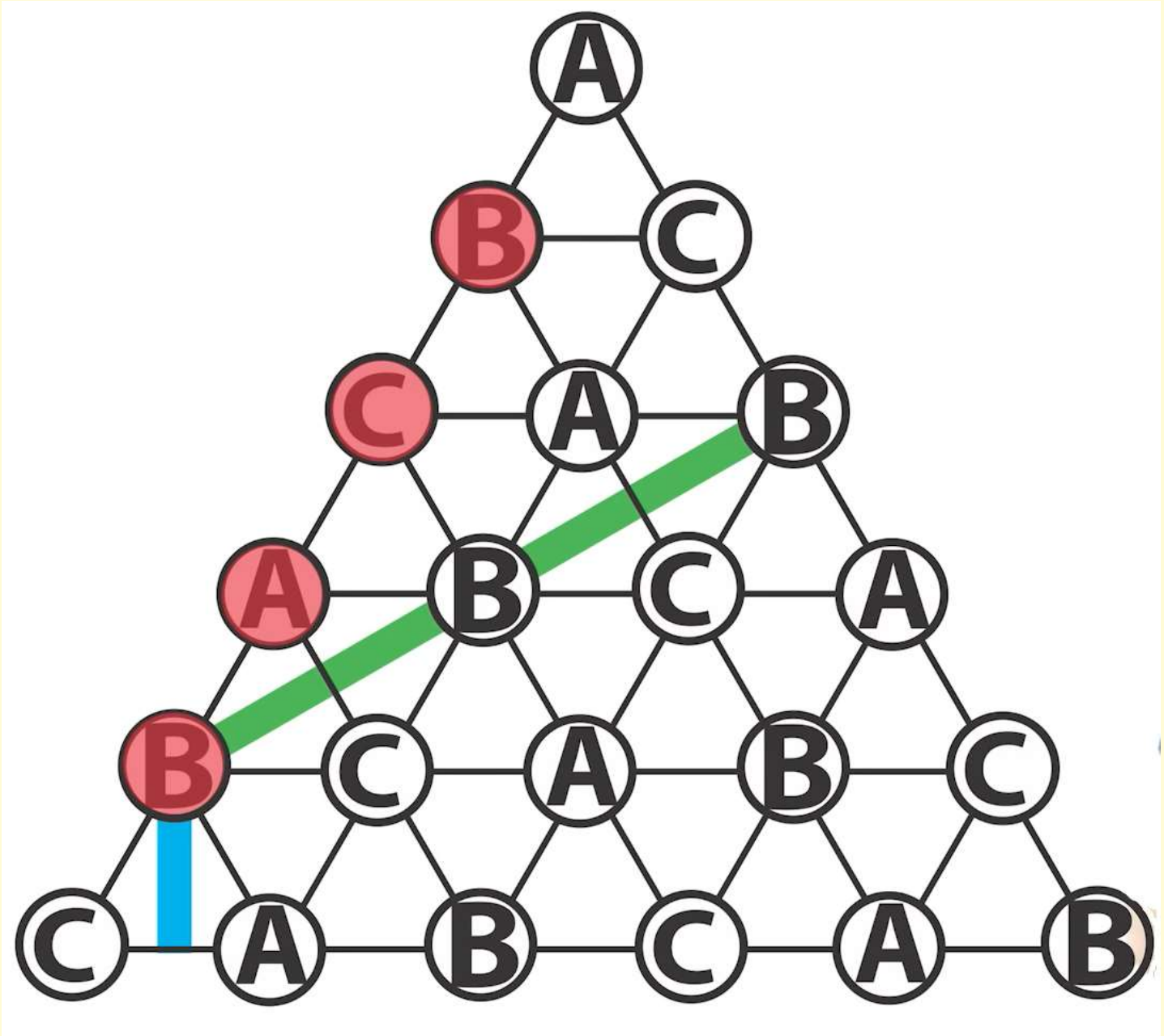
$0+400+100=500$



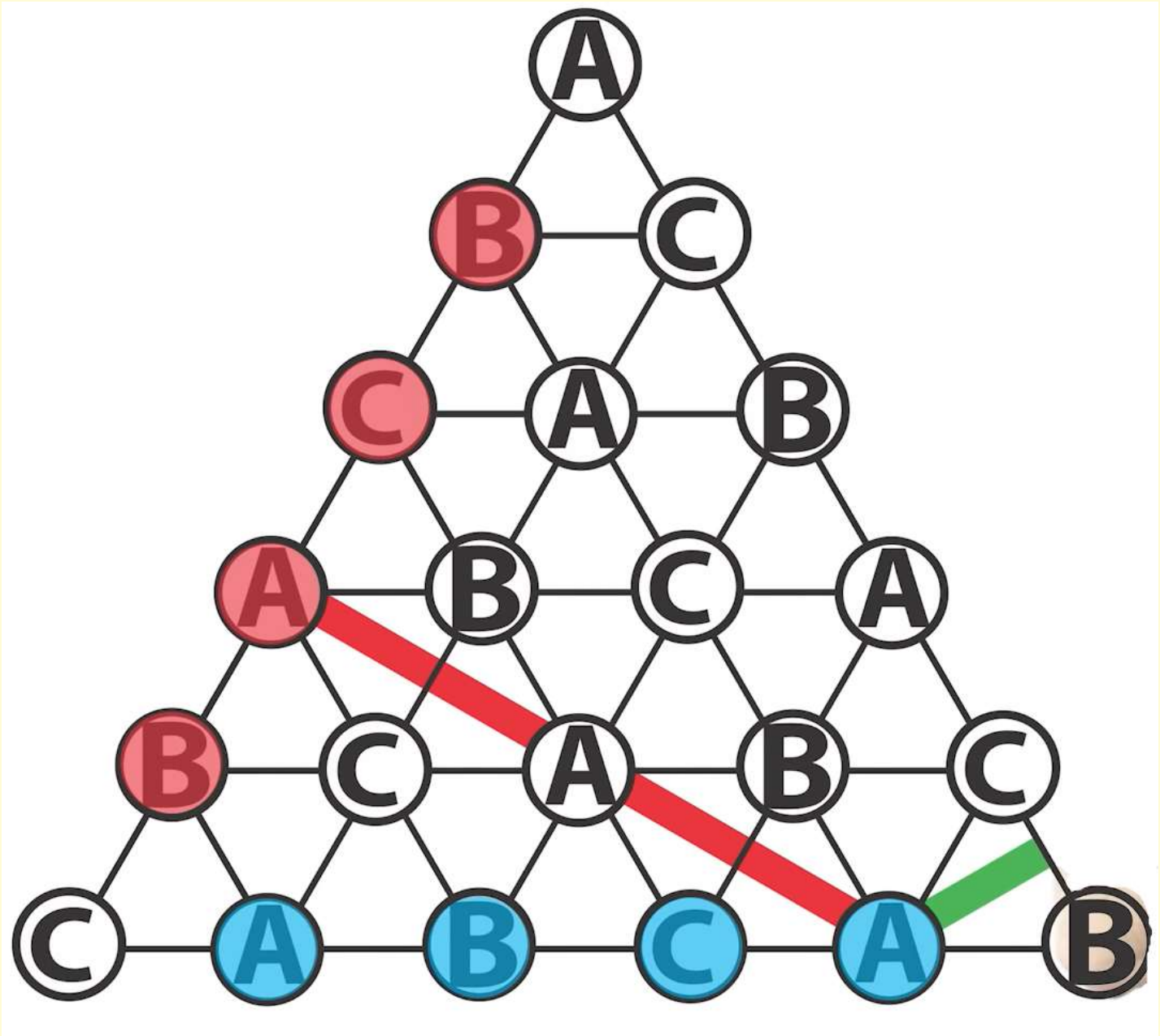
$0+300+200=500$



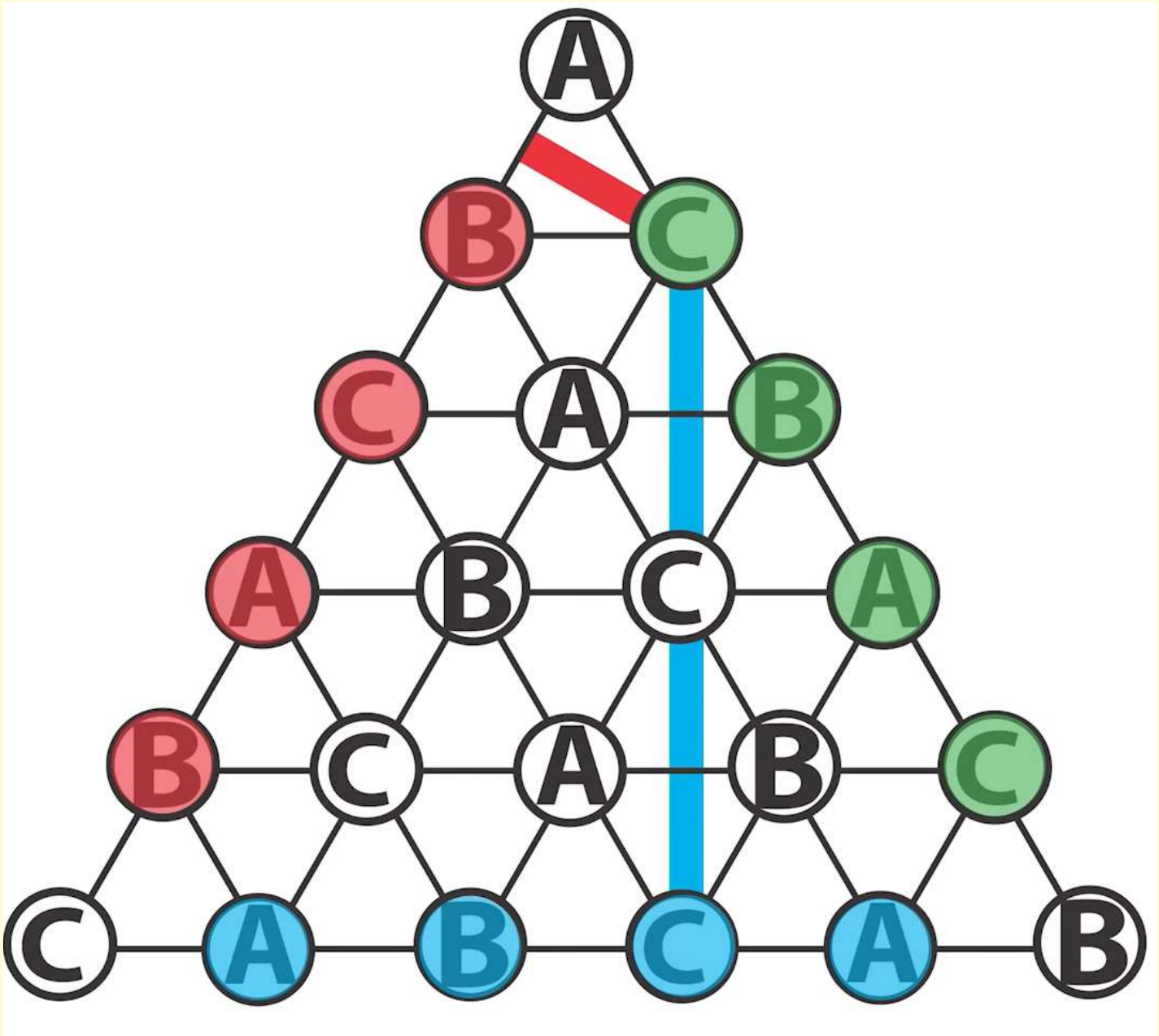
$0+100+400=500$



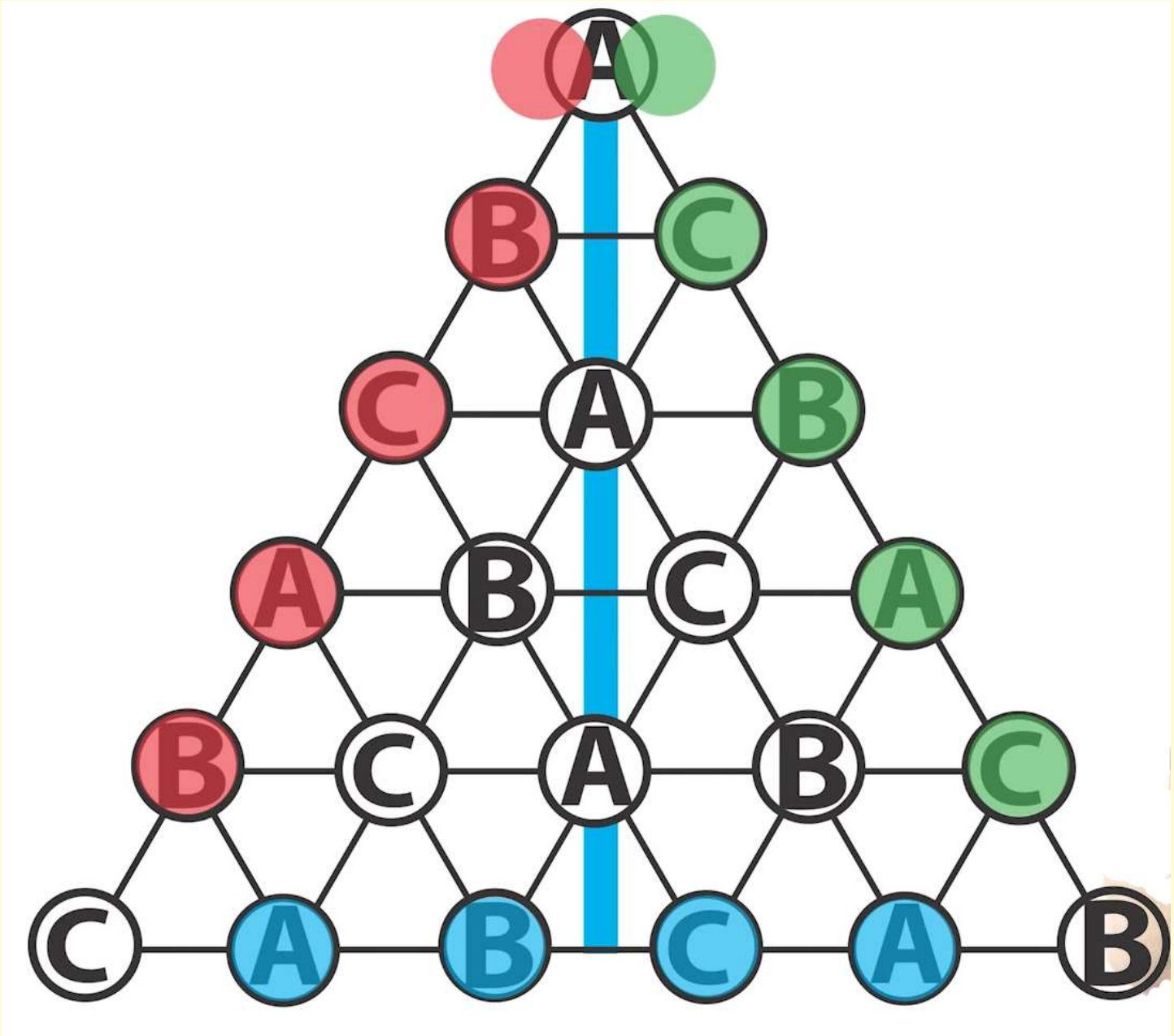
400+0+100=500



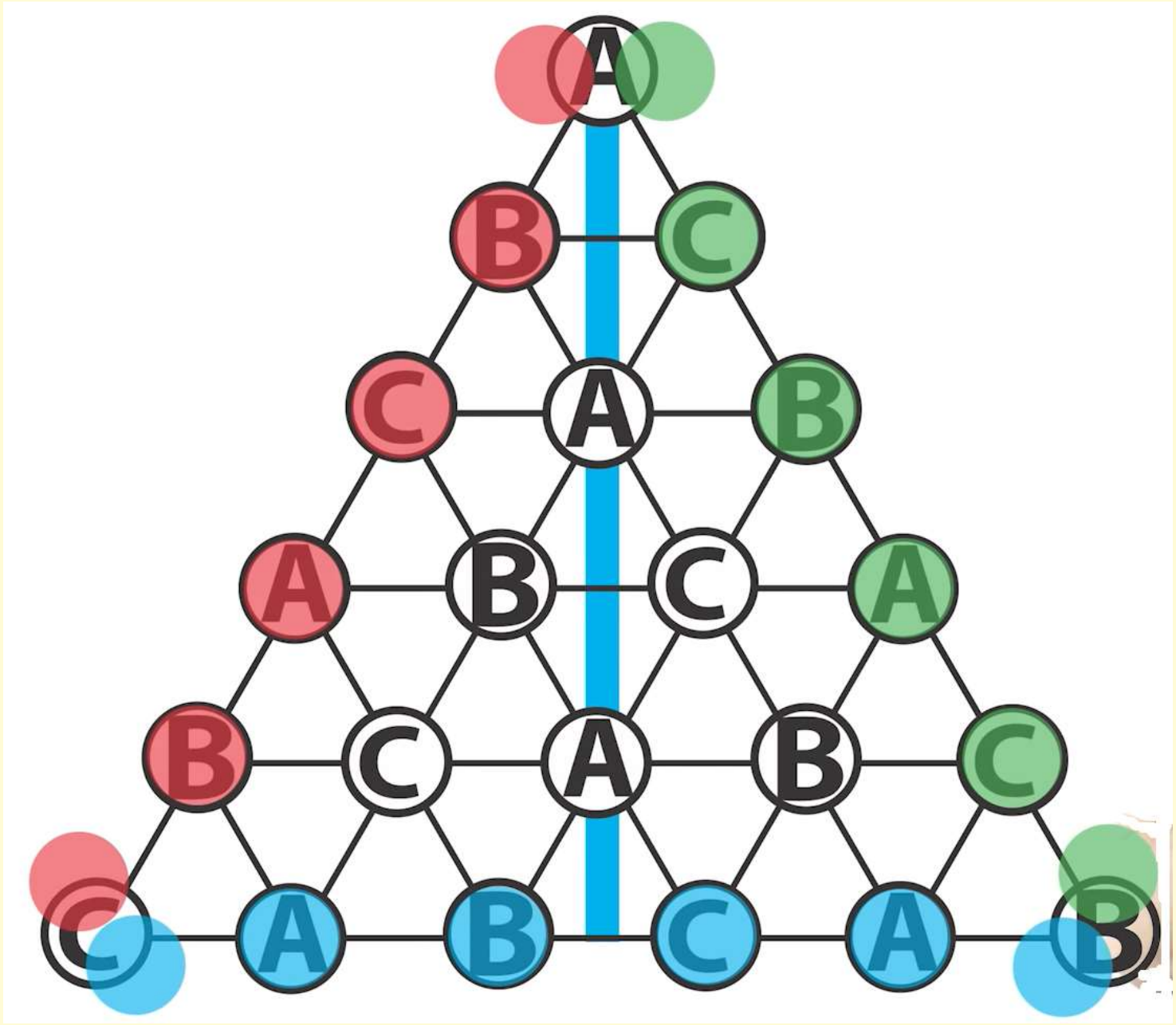
100+400+0=500



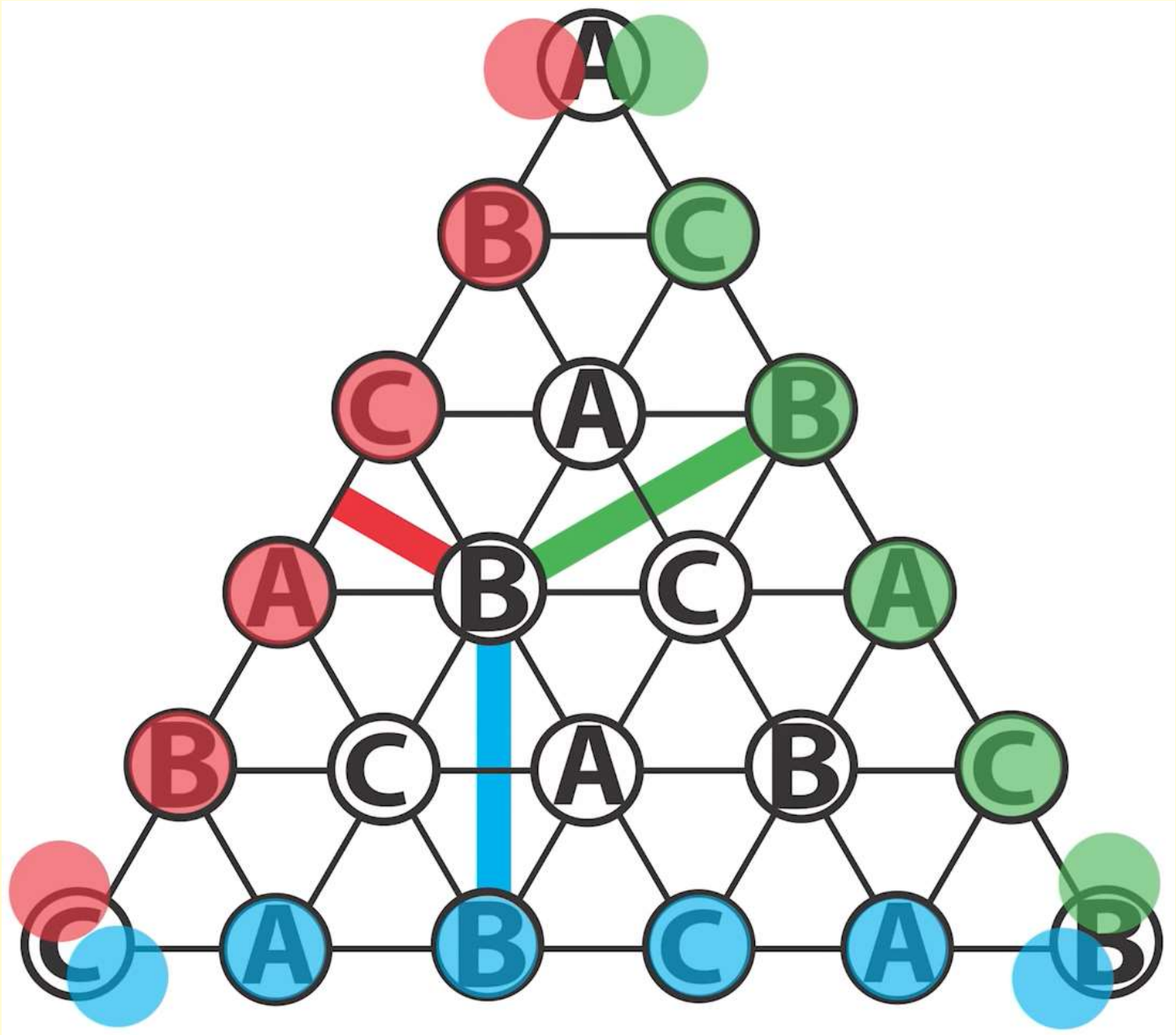
$0+500+0=500$



$0+500+0=500$



$100+200+200=500$



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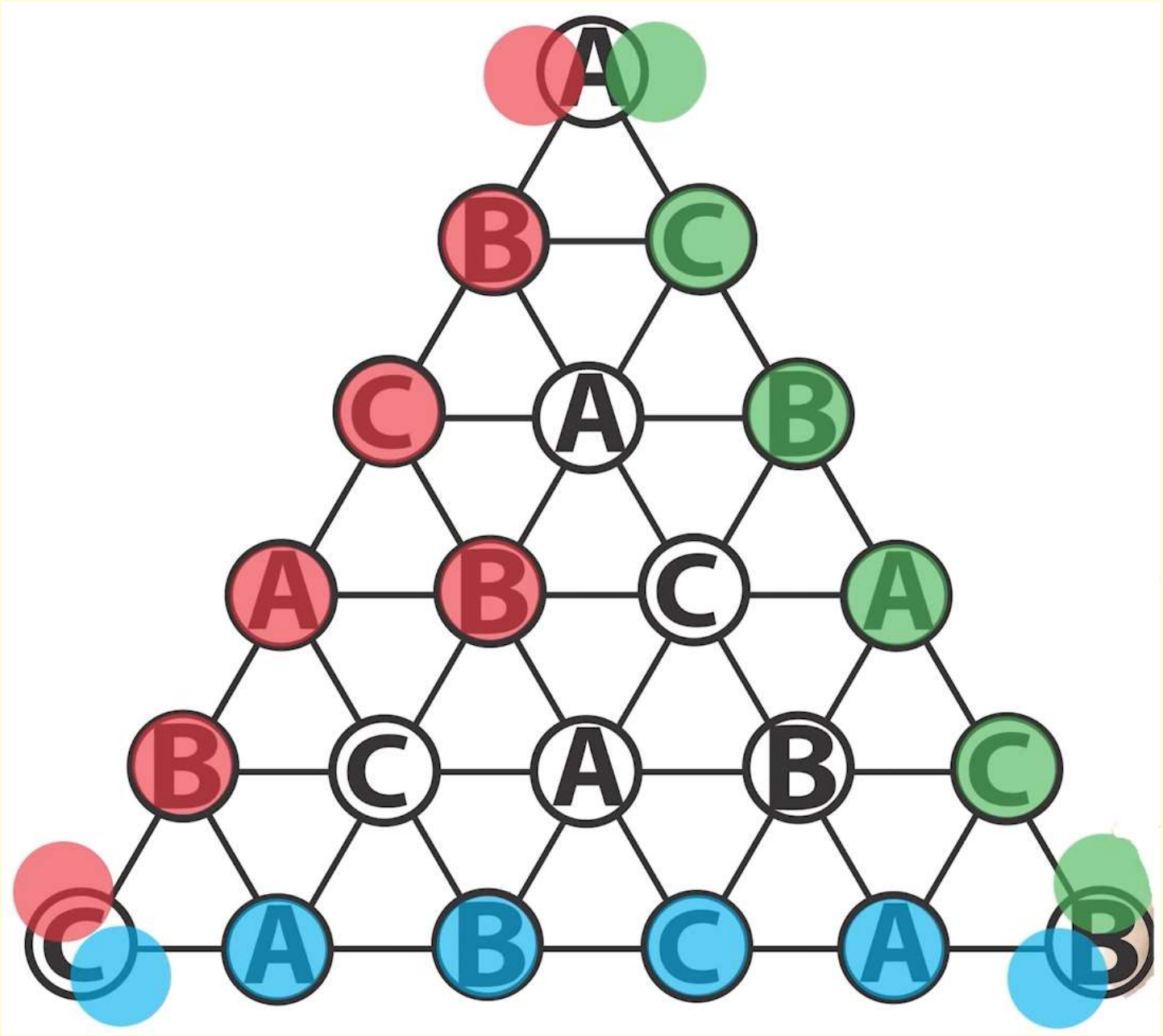
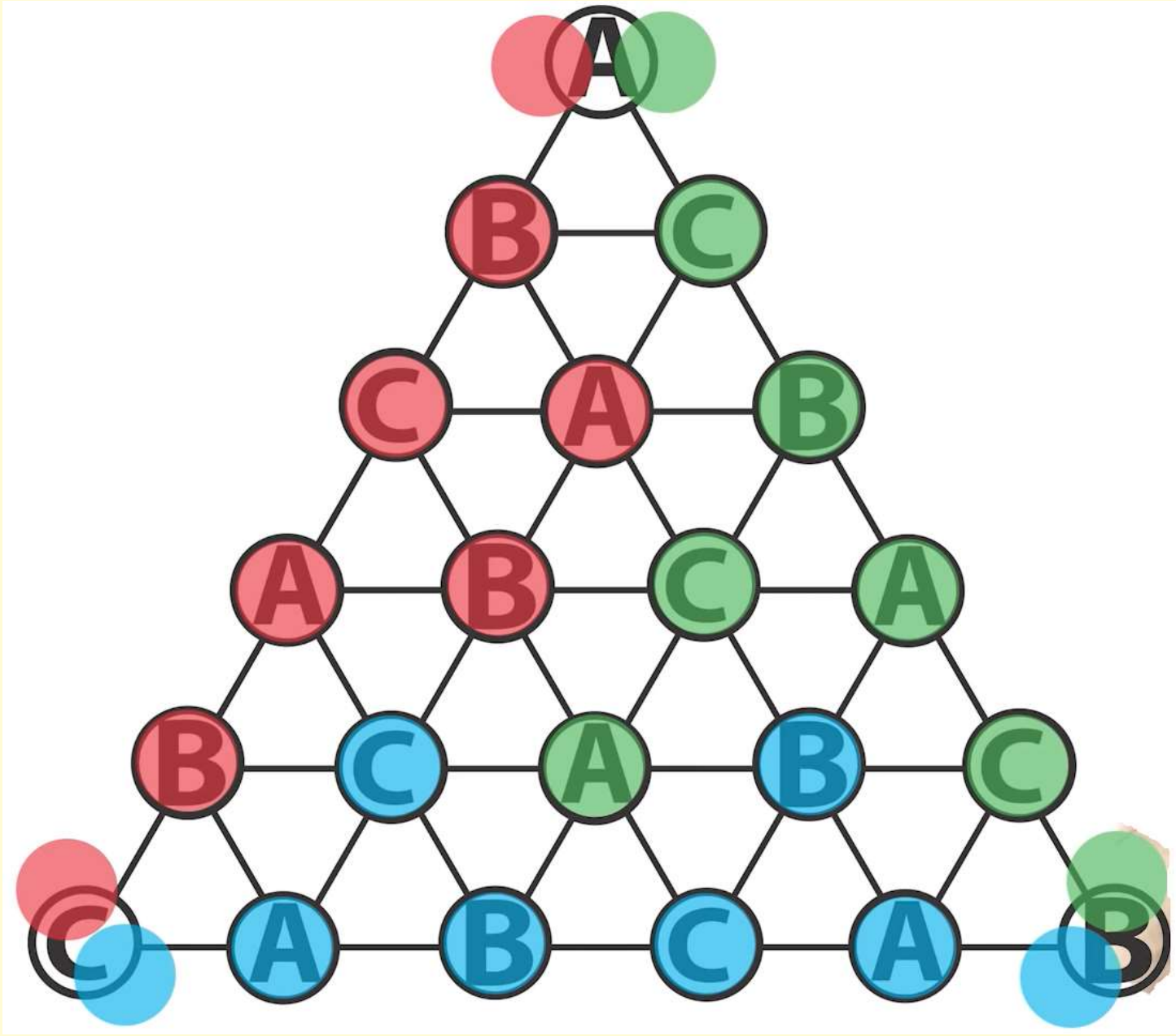
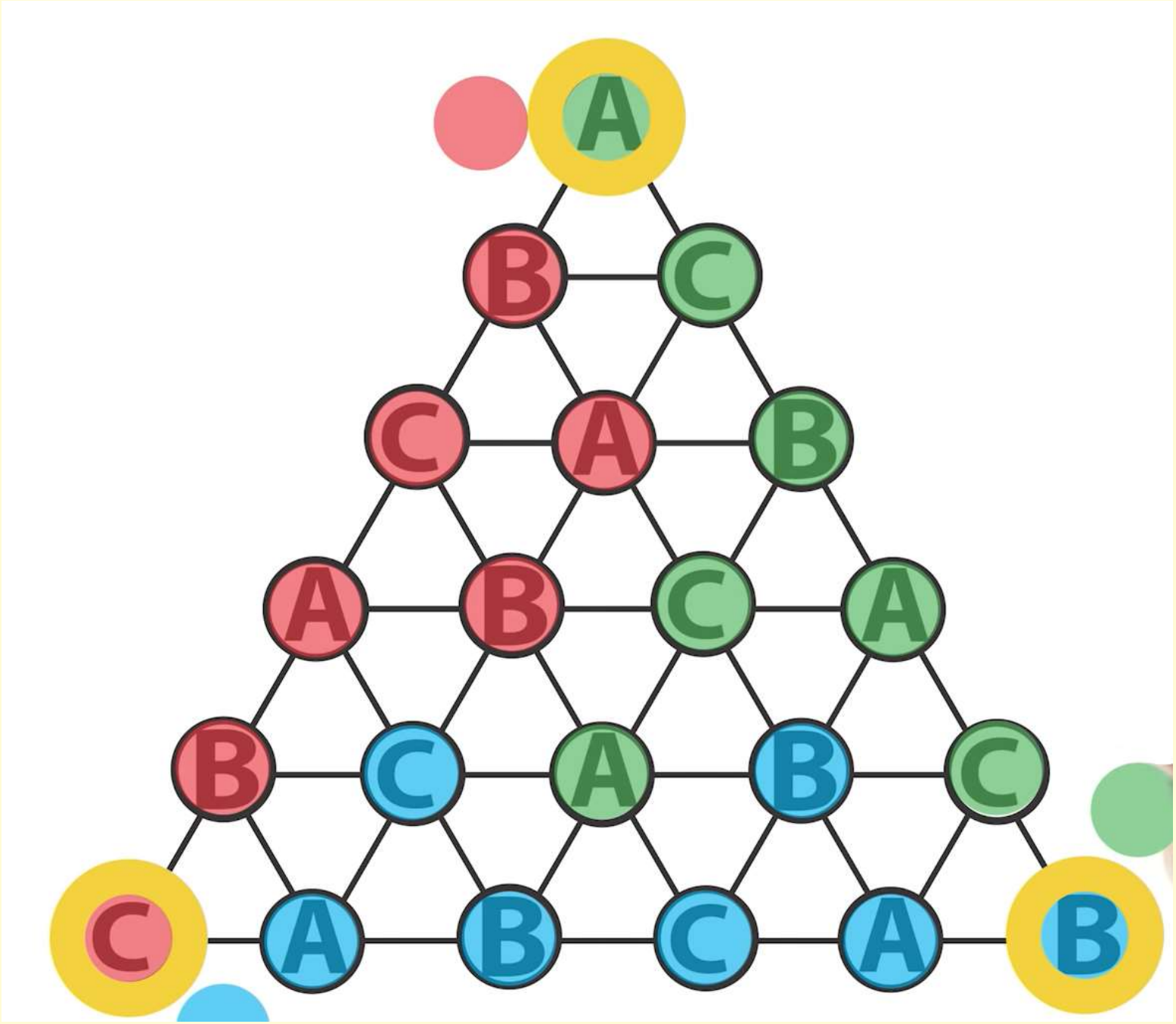


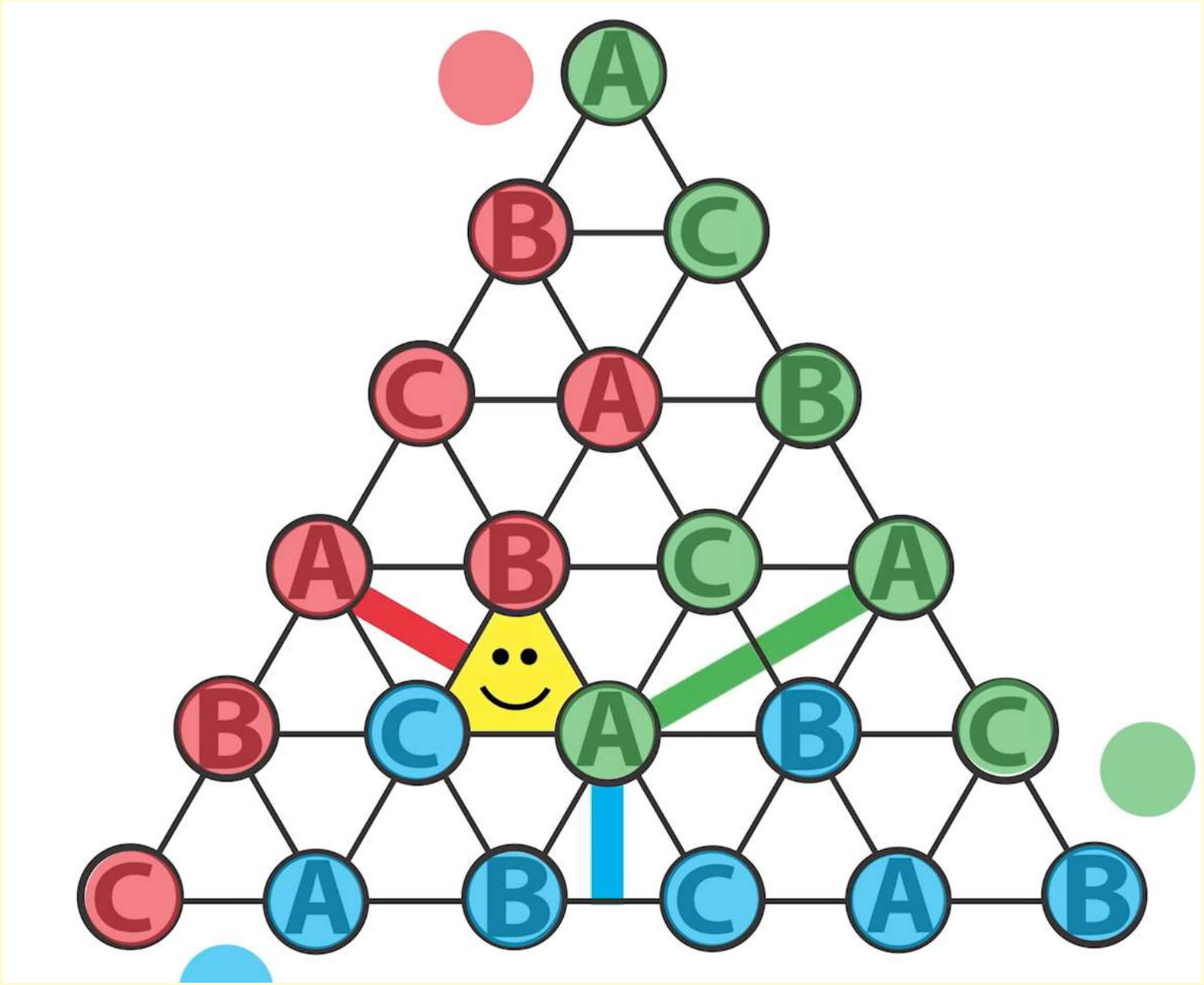
fill the rest similarly by asking the choices



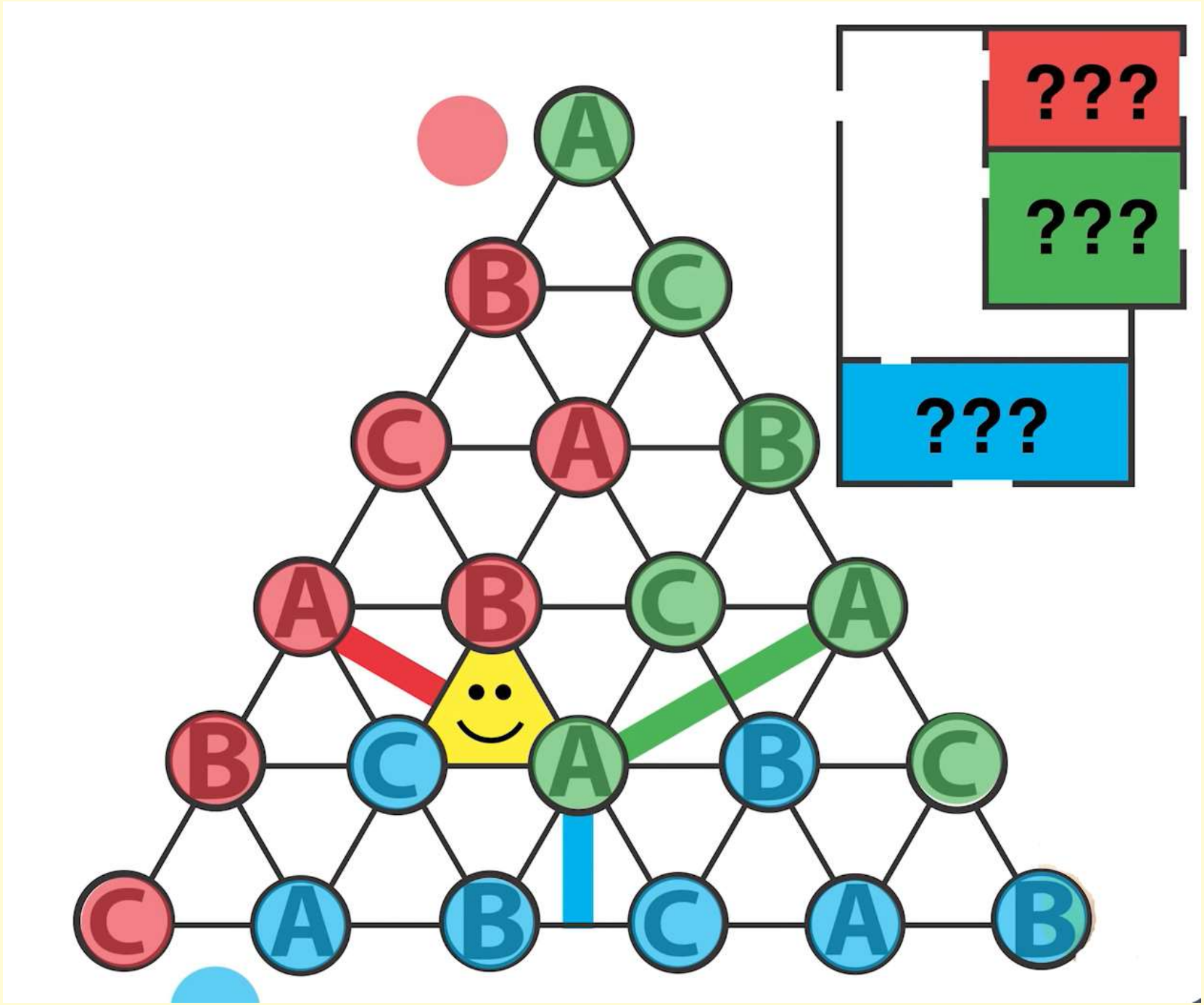
assume distinct choices at the corners



?+?+?=500



00+00+00=500



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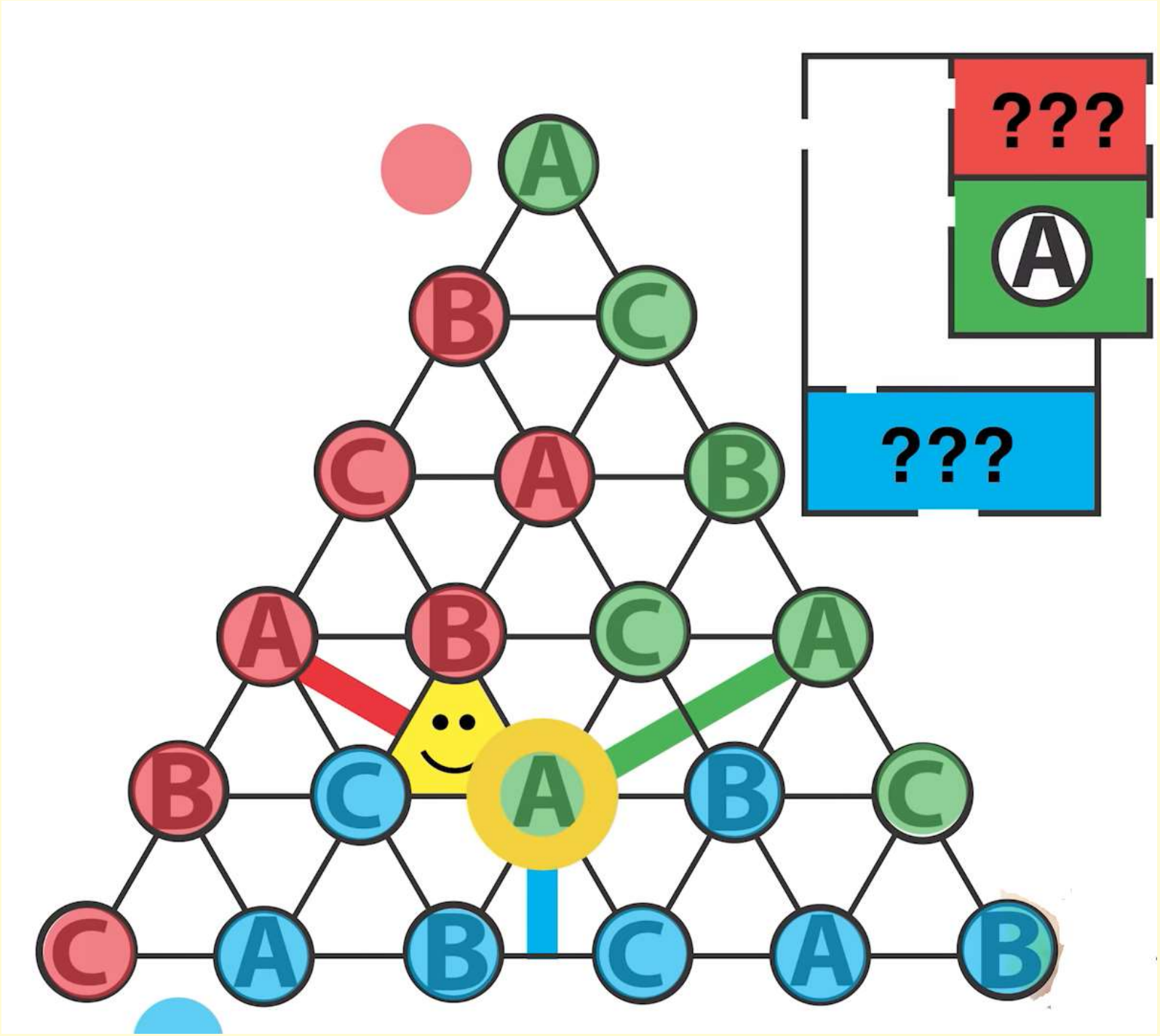

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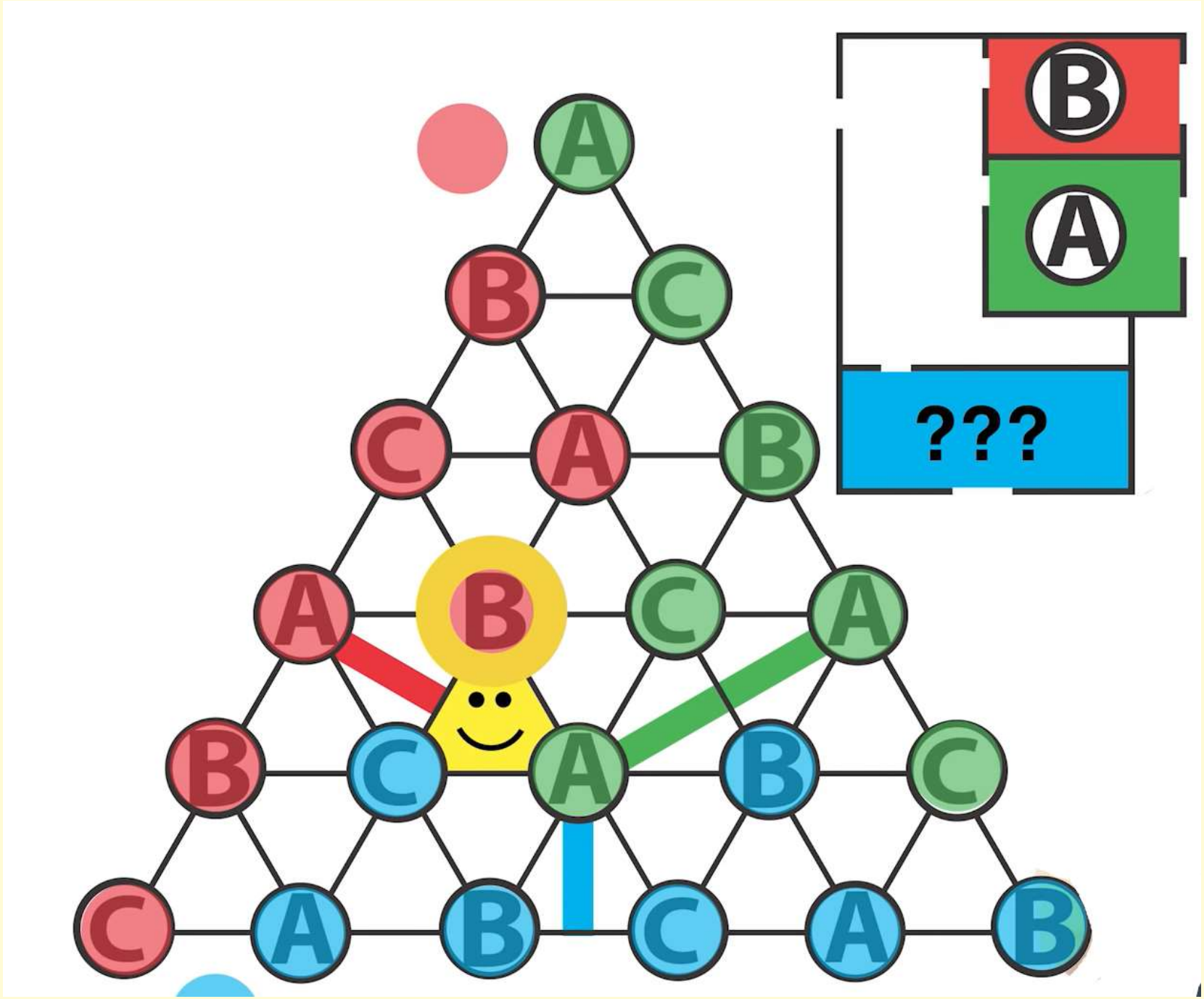

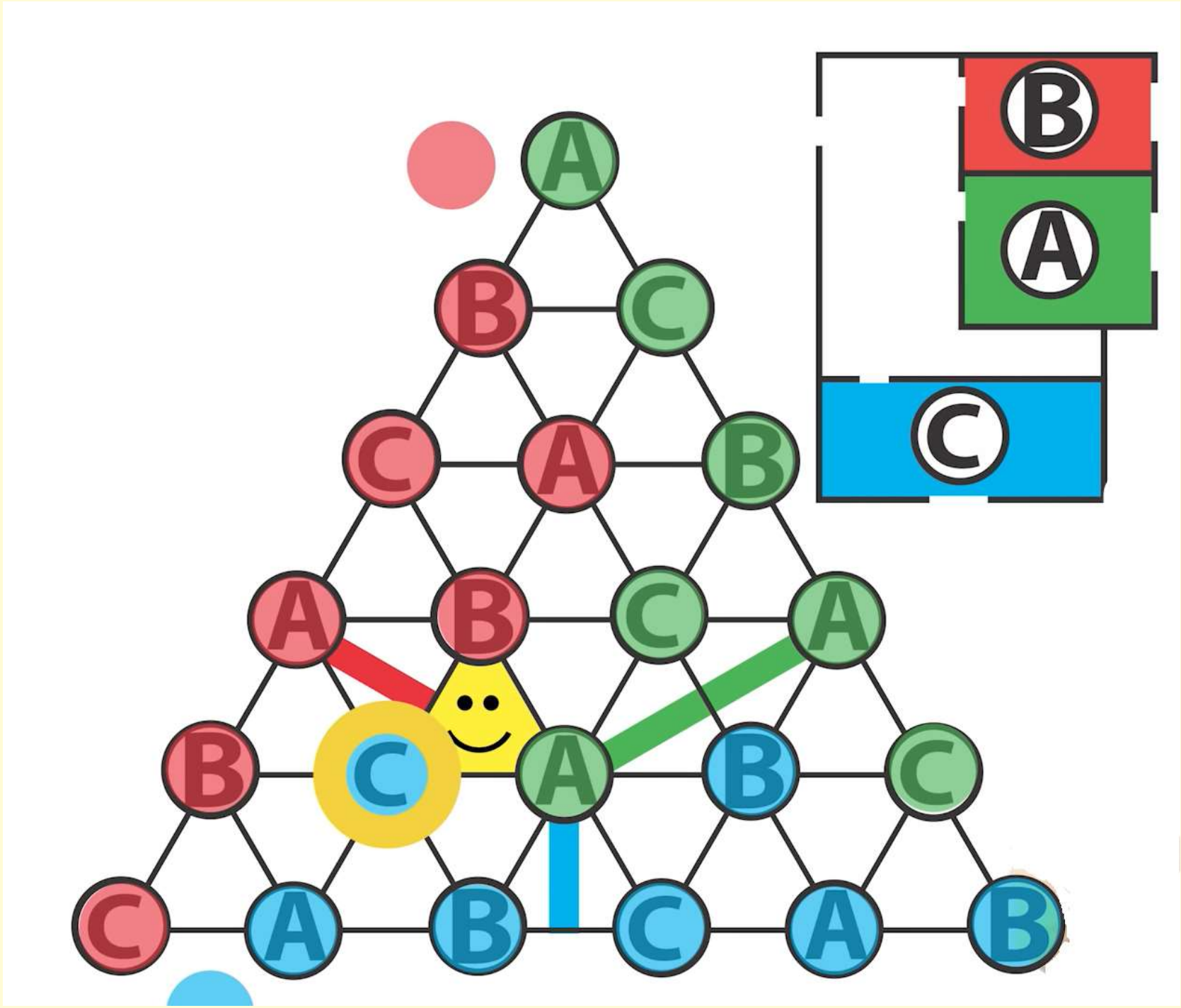
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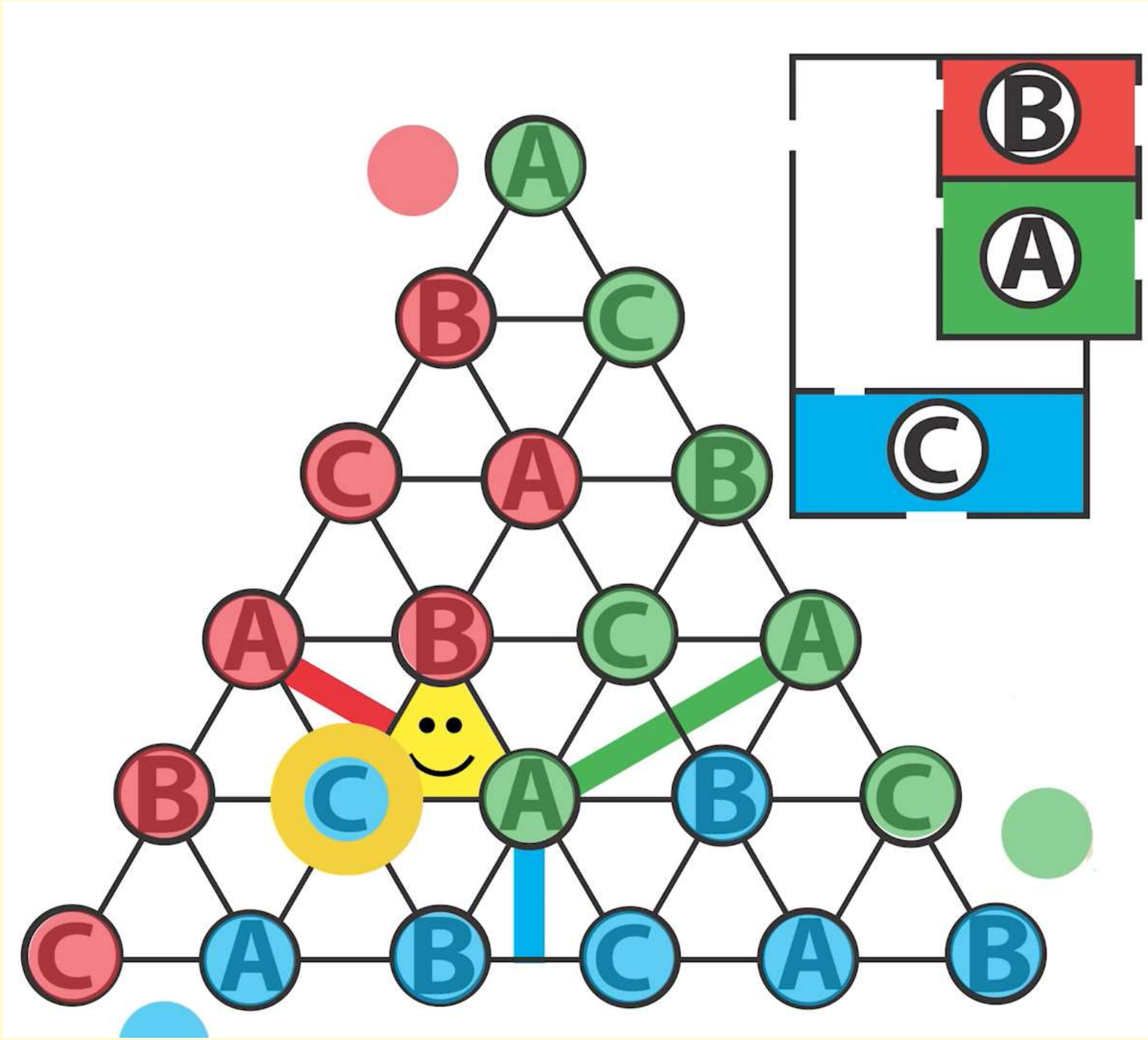
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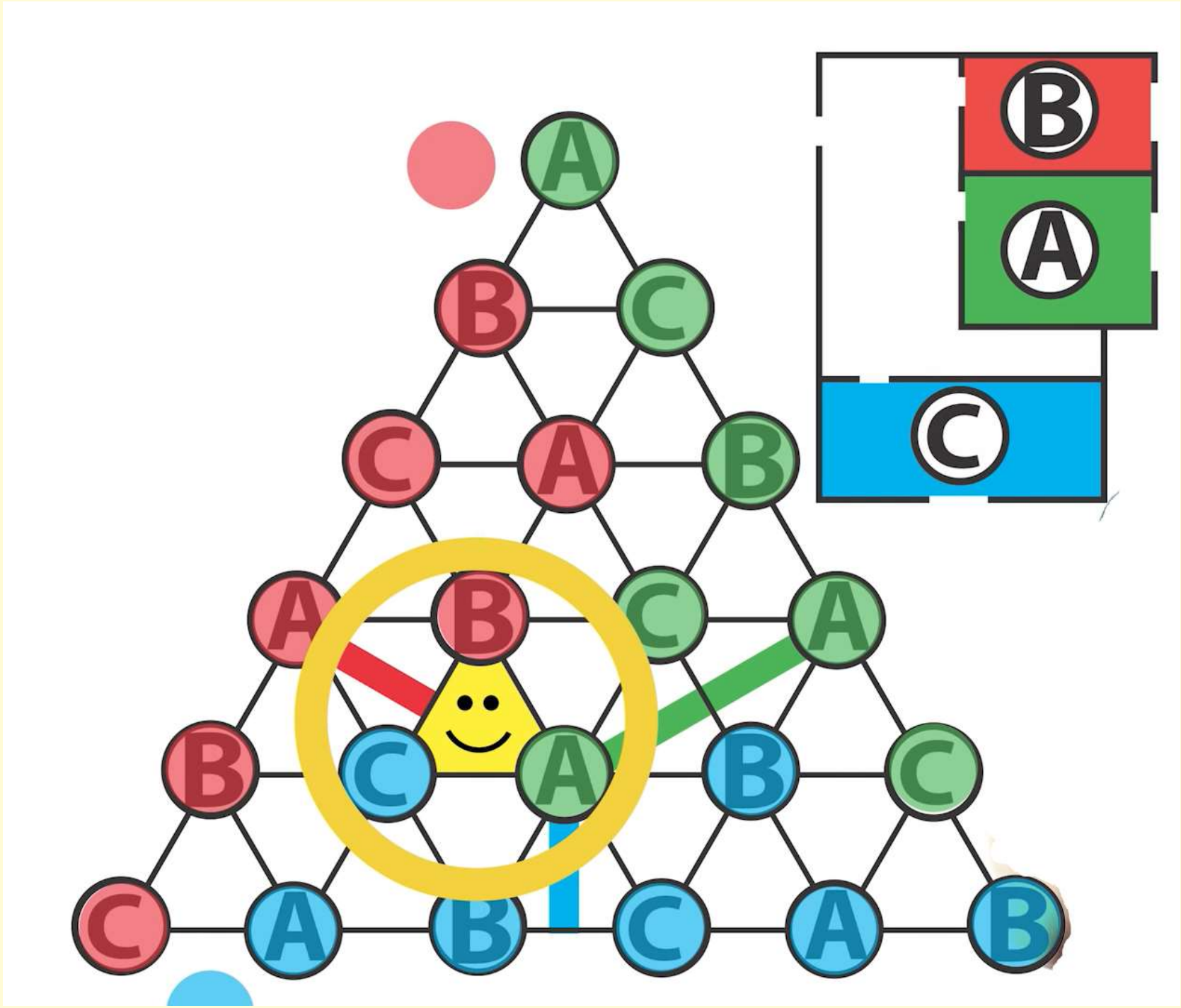
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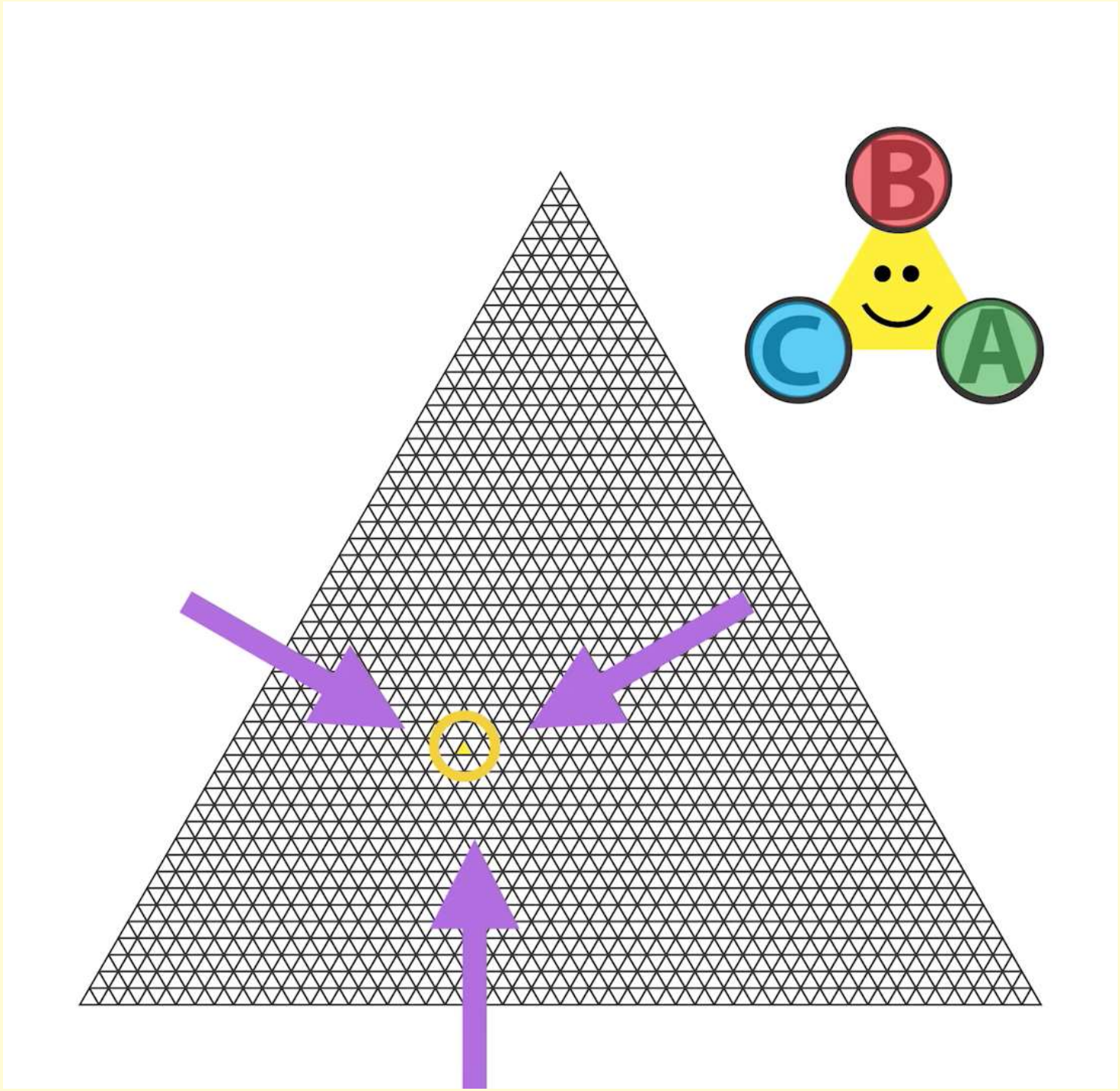
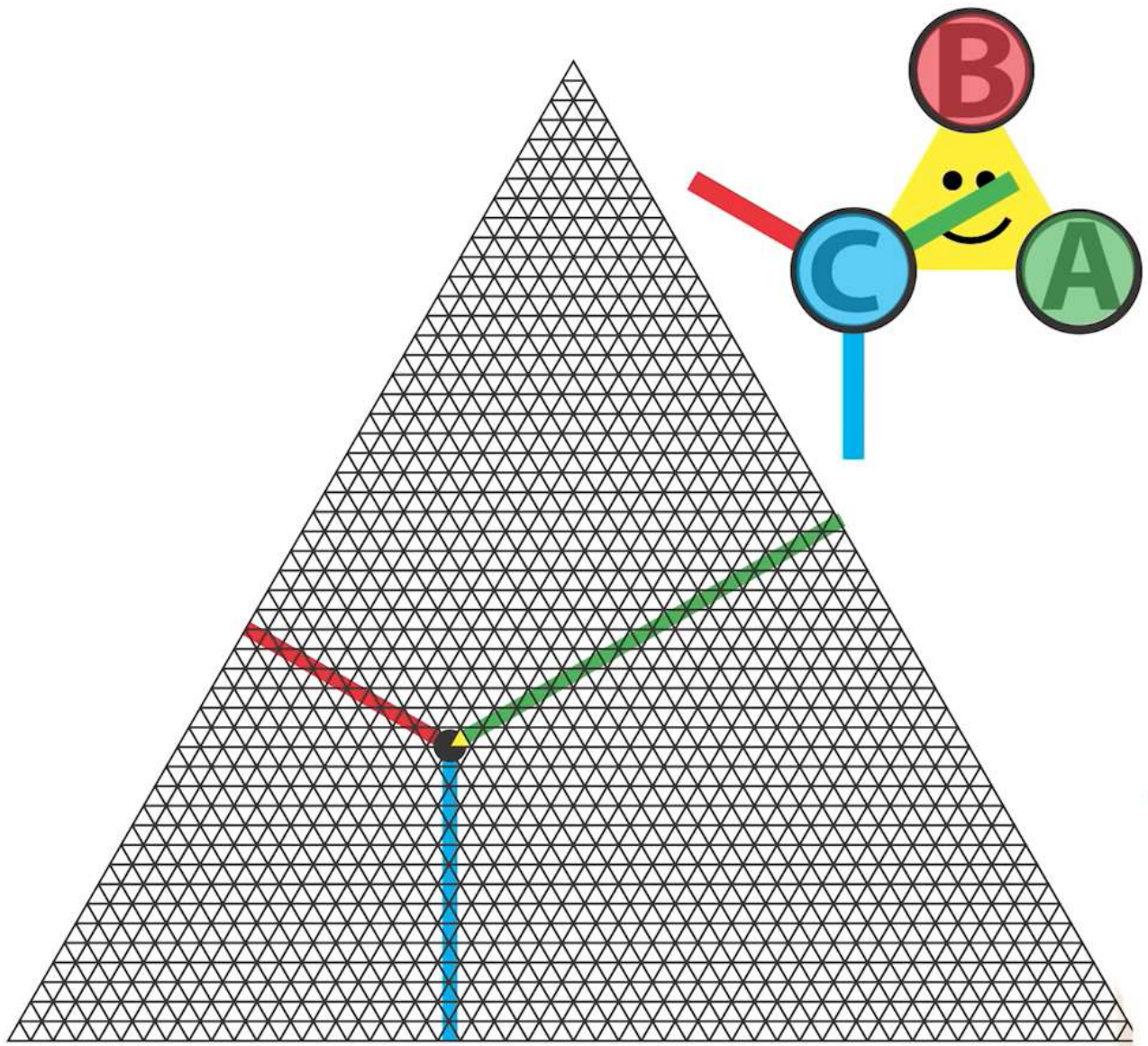
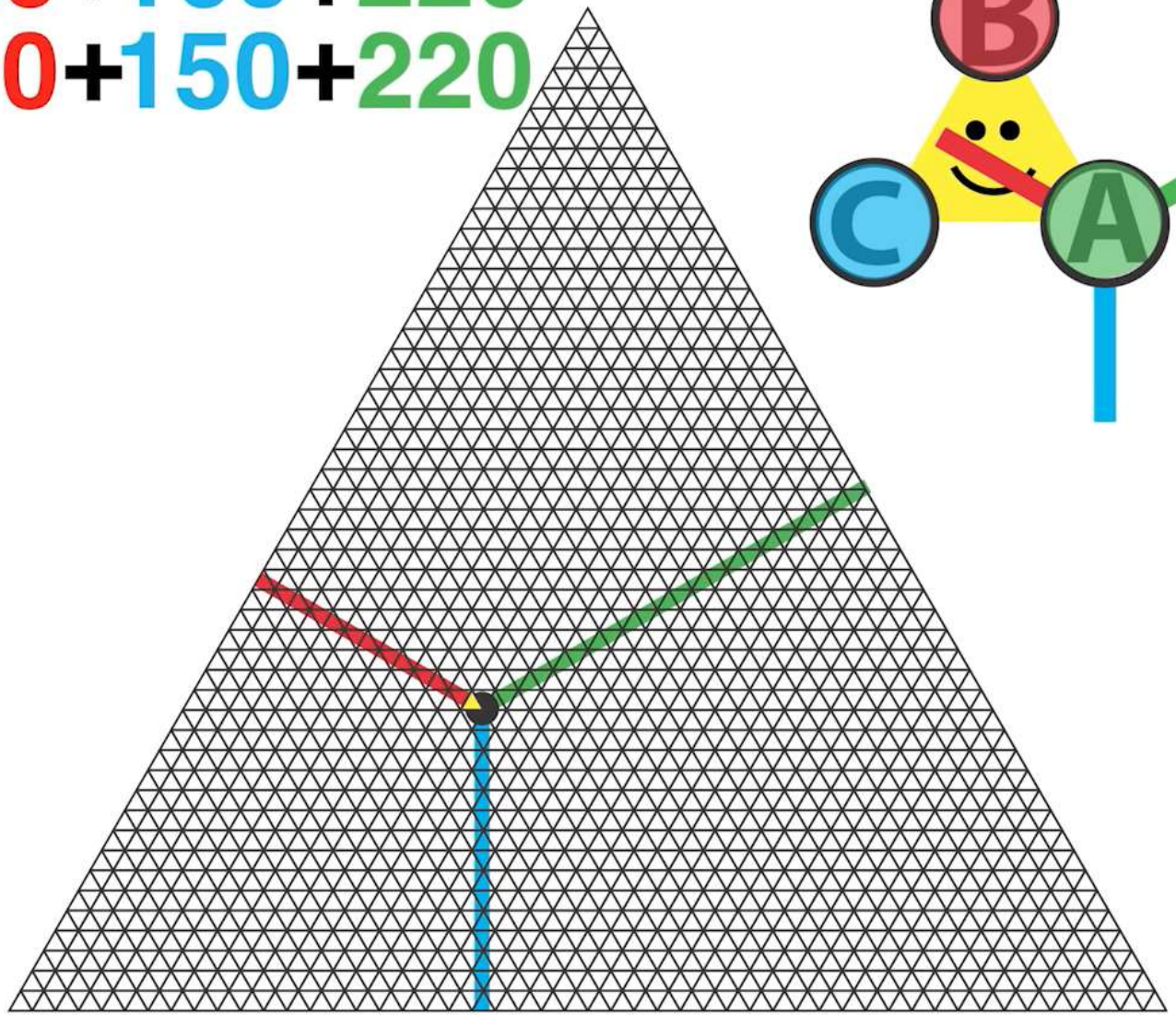
A
C

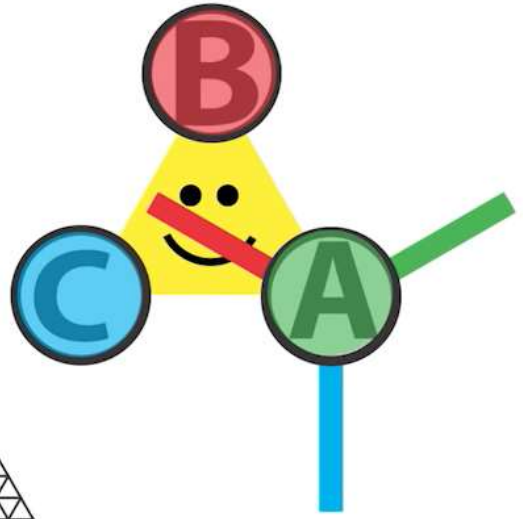

B
C
A

120+150+230



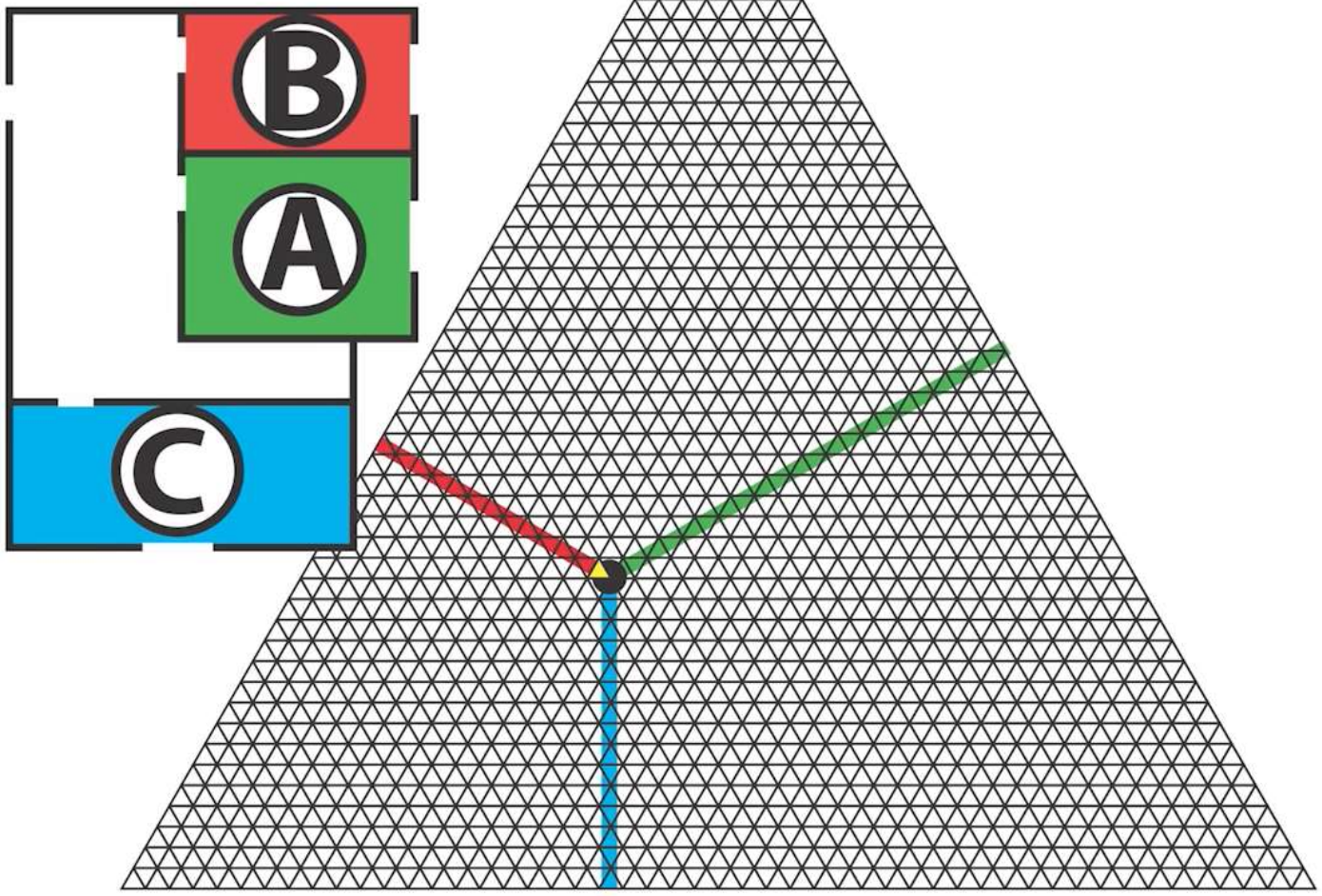
120+150+230
120+160+220
130+150+220



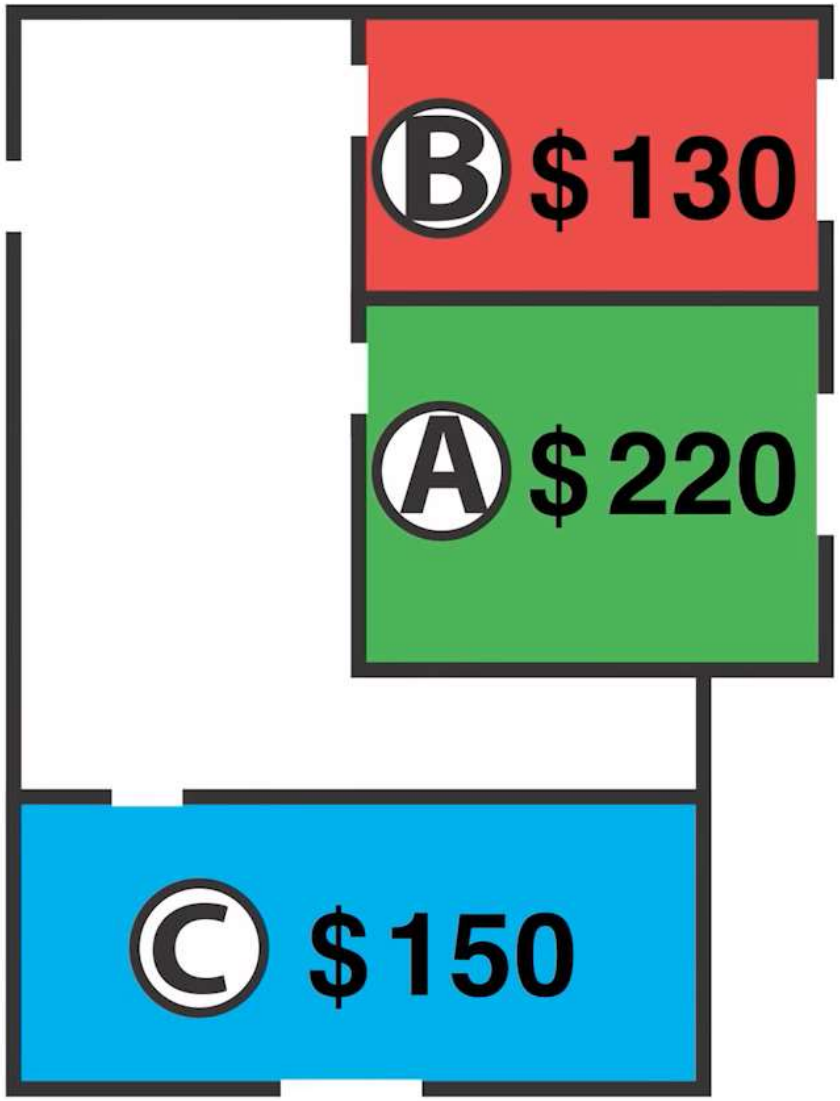


120+150+230
120+160+220
130+150+220

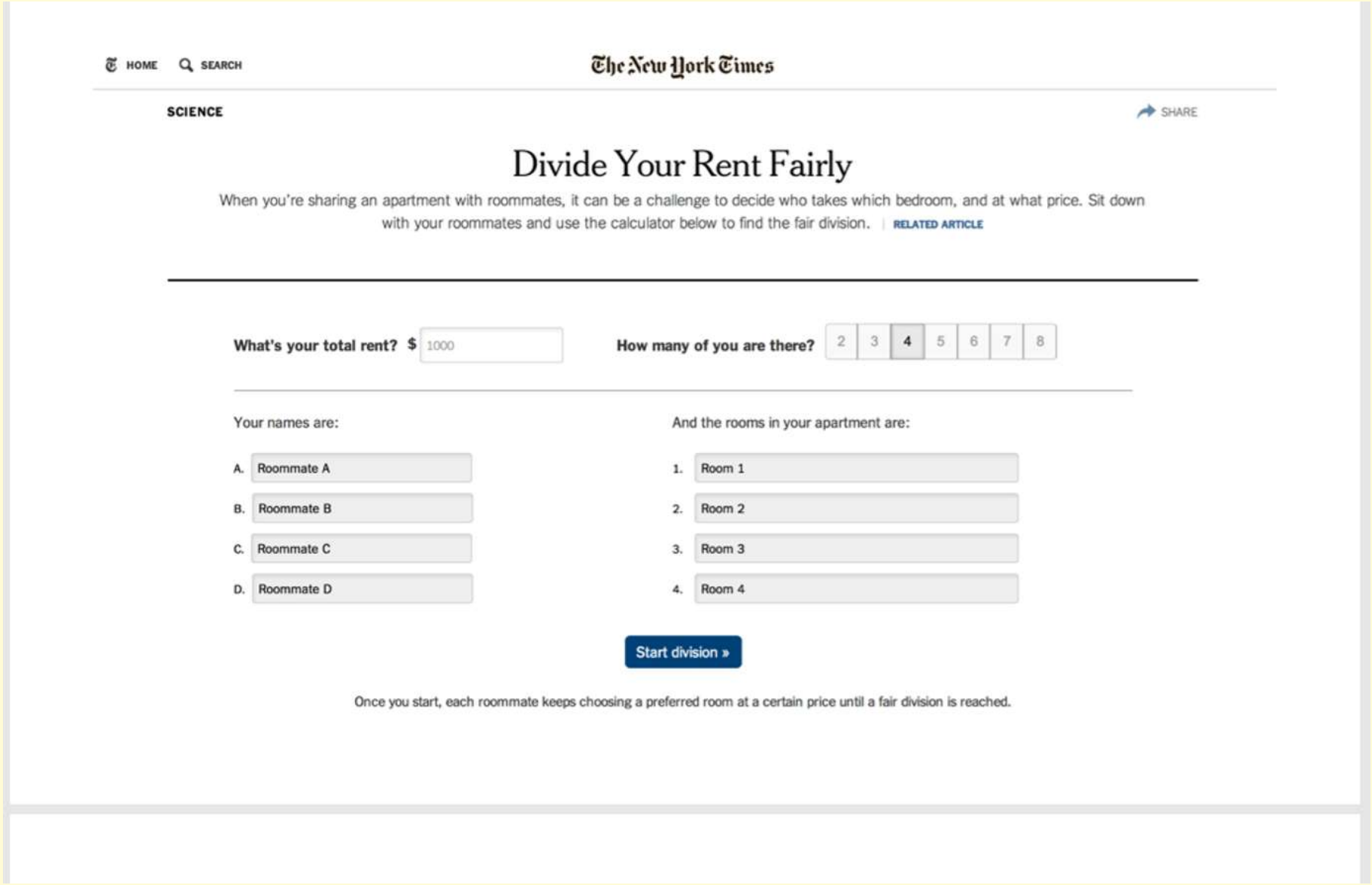
NEGLIGIBLE $10 DIFFERENCES



a fair allocation



is it practical?



HOME SEARCH

The New York Times

SCIENCE

SHARE

Divide Your Rent Fairly

When you're sharing an apartment with roommates, it can be a challenge to decide who takes which bedroom, and at what price. Sit down with your roommates and use the calculator below to find the fair division. | RELATED ARTICLE

What's your total rent? $ 1000

How many of you are there? 2 3 4 5 6 7 8

Your names are:

A. Roommate A
B. Roommate B
C. Roommate C
D. Roommate D

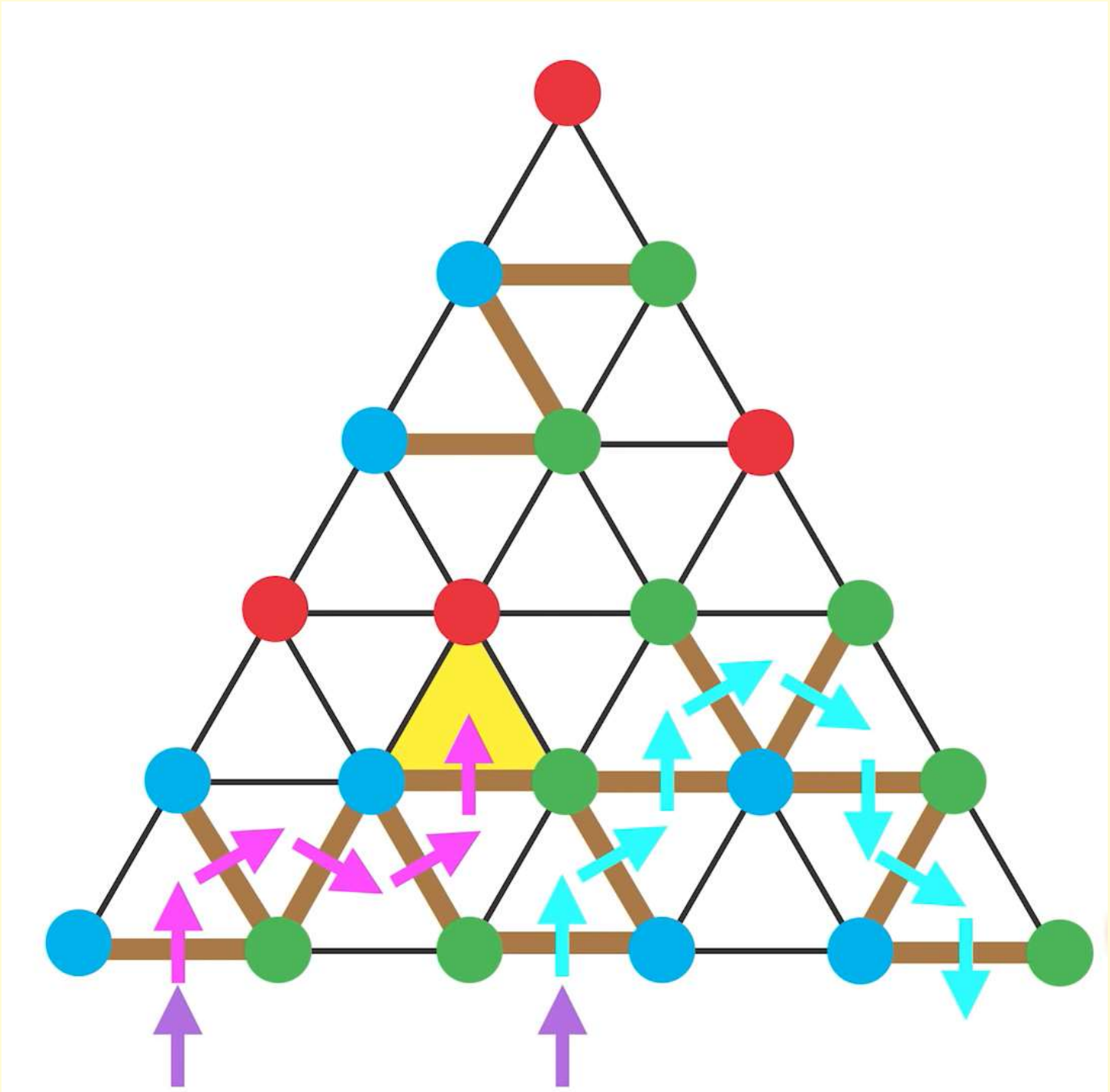
And the rooms in your apartment are:

1. Room 1
2. Room 2
3. Room 3
4. Room 4

Start division »

Once you start, each roommate keeps choosing a preferred room at a certain price until a fair division is reached.

is it practical?



Pictures are from