Foundations of self-progressive choice models

KEMAL YILDIZ
Bilkent University & Princeton University

a snapshot

- random choice models are used successfully to identify heterogeneity in aggregate choice behavior

a snapshot

- random choice models are used successfully to identify heterogeneity in aggregate choice behavior
- despite prominent choice models, such as the RUM, are underidentified: multiple representations

a snapshot

- random choice models are used successfully to identify heterogeneity in aggregate choice behavior
- despite prominent choice models, such as the RUM, are underidentified: multiple representations
- panacea has been adding structure into the model to obtain a unique representation.

a snapshot

- random choice models are used successfully to identify heterogeneity in aggregate choice behavior
- despite prominent choice models, such as the RUM, are underidentified: multiple representations
- panacea has been adding structure into the model to obtain a unique representation.

e.g. RUM$\to$ probit, logit (Luce rule)

Here, instead of focusing on a specific choice model, we present a complementary approach:

our approach

- we take choice models as the primitive objects, and

our approach

- we take choice models as the primitive objects, and
- assume an “orderliness” in the population (e.g. risk attitudes) that allows for partial comparison of agents’ choice behaviors, thus derives the heterogeneity.

our approach

- we take choice models as the primitive objects, and
- assume an “orderliness” in the population (e.g. risk attitudes) that allows for partial comparison of agents’ choice behaviors, thus derives the heterogeneity.

We propose and analyze self-progressive choice models

our approach

- we take choice models as the primitive objects, and
- assume an “orderliness” in the population (e.g. risk attitudes) that allows for partial comparison of agents’ choice behaviors, thus derives the heterogeneity.

We propose and analyze self-progressive choice models that provide for unique orderly representation for each aggregate (random) choice behavior consistent with the model.

motivation

- using a self-progressive choice model would facilitate organization and analysis of aggregate (random) choice data for an analyst

motivation

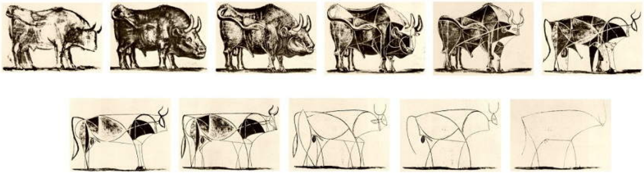
- using a self-progressive choice model would facilitate organization and analysis of aggregate (random) choice data for an analyst

motivation

- using a self-progressive choice model would facilitate organization and analysis of aggregate (random) choice data for an analyst

- who seeks to describe the population heterogeneity derived from a given ordering.

Self-progressive choice models



outline 

A **self-progressive** choice model provides for a unique orderly representation for each aggregate (random) choice behavior consistent with the model.

components:

I. (deterministic) choice models

II. orderliness

III. random choice models

I. deterministic choice model

X is an alternative set with n elements

I. deterministic choice model

X is an alternative set with n elements

choice sets are nonempty $S \subset X$

I. deterministic choice model

X is an alternative set with n elements

choice sets are nonempty $S \subset X$

choice space is a collection of choice sets: Ω
(limited observations are allowed)

I. deterministic choice model

X is an alternative set with n elements

choice sets are nonempty $S \subset X$

choice space is a collection of choice sets: Ω
(limited observations are allowed)

a choice function c singles out an alternative from each $S \in \Omega$.

I. deterministic choice model

X is an alternative set with n elements

choice sets are nonempty $S \subset X$

choice space is a collection of choice sets: Ω
(limited observations are allowed)

a choice function c singles out an alternative from each $S \in \Omega$.

a choice model is a set of choice functions: μ

I. deterministic choice model

X is an alternative set with n elements

choice sets are nonempty $S \subset X$

choice space is a collection of choice sets: Ω
(limited observations are allowed)

a choice function c singles out an alternative from each $S \in \Omega$.

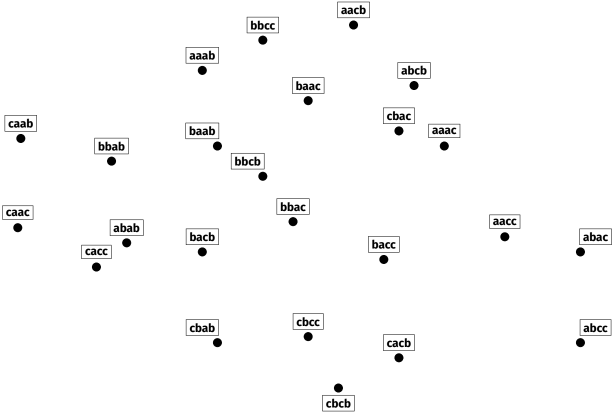
a choice model is a set of choice functions: μ

μ specifies which choice behaviors are admissible.

e.g. rational model: choice functions maximizing a preference

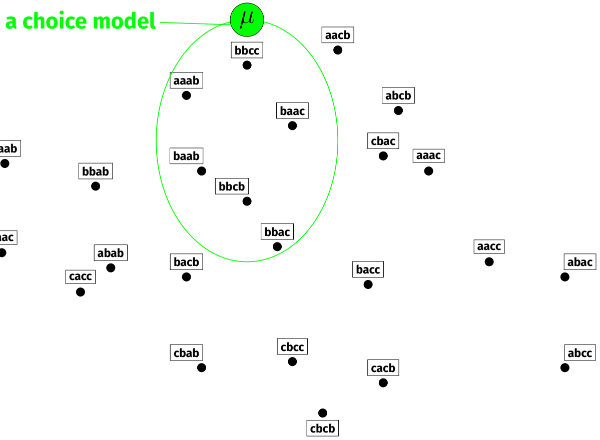
example

- Let $X = \{a, b, c\}$ & $\Omega = \{abc, ab, ac, bc\}$
- a choice function $c = [\, a \quad a \quad c \quad b \,]$



example

- Let $X = \{a, b, c\}$ & $\Omega = \{abc, ab, ac, bc\}$
- a choice function $c = [\, a \quad a \quad c \quad b \,]$



II. "orderliness"

- a **primitive ordering** $>$ is a complete, transitive, & antisymmetric binary relation over X ($>$: $a > b > c$)

  e.g. objective values/rational assessment, risk or time prefs.

- a is "**better than**" ($\geq$) b: means $a > b$ or $a = b$.

II. “orderliness”

- a **primitive ordering** $>$ is a complete, transitive, & antisymmetric binary relation over X ($>$: $a > b > c$)

  e.g. objective values/rational assessment, risk or time prefs.
- a is “**better than**” ($\geq$) b: means $a > b$ or $a = b$.

*We induce a **domination relation** $\rhd$ to **compare** different choice functions from the primitive ordering $>$ s.t.*

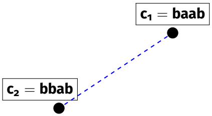
$$c \rhd c' \text{ iff } c(S) \geq c'(S) \text{ for every } S \in \Omega$$

from primitive ordering $a > b > c$ to $\triangleright$

- c dominates c'–denoted by $c \triangleright c'$–iff for every S,

$$c(S) > c'(S) \text{ or } c(S) = c'(S).$$

Ω	c_1	c_2
$\{a, b, c\}$	b	b
$\{a, b\}$	a	b
$\{a, c\}$	a	a
$\{b, c\}$	b	b

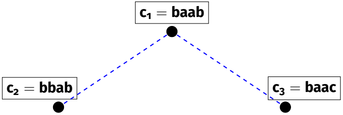


from primitive ordering $a > b > c$ to $\rhd$

- c dominates c'–denoted by $c \rhd c'$–iff for every S,

$$c(S) > c'(S) \text{ or } c(S) = c'(S).$$

Ω	c_1	c_2	c_3
$\{a, b, c\}$	b	b	b
$\{a, b\}$	a	b	a
$\{a, c\}$	a	a	a
$\{b, c\}$	b	b	c

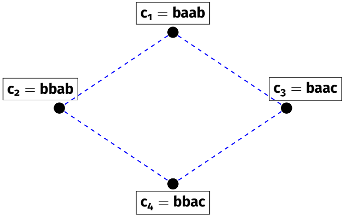


from primitive ordering $a > b > c$ to $\triangleright$

- c **dominates** c'–denoted by $c \triangleright c'$–iff for every S,

$$c(S) > c'(S) \text{ or } c(S) = c'(S).$$

Ω	c_1	c_2	c_3	c_4
$\{a, b, c\}$	b	b	b	b
$\{a, b\}$	a	b	a	b
$\{a, c\}$	a	a	a	a
$\{b, c\}$	b	b	c	c



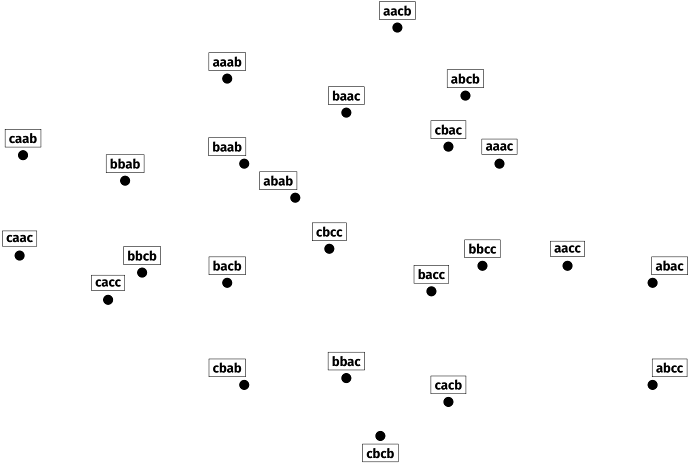


Figure: set of all choice functions

orderliness: $a > b > c \rightarrow \triangleright$

orderliness: $a > b > c \to \triangleright$

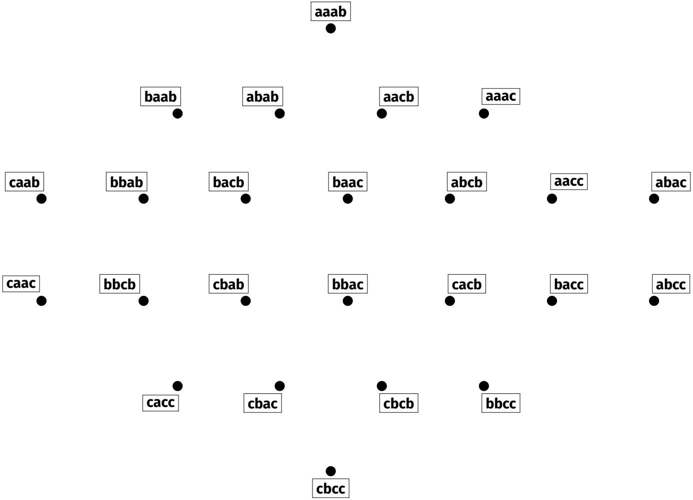


Figure: set of choice functions ordered wrt $\triangleright$.

orderliness: $a > b > c \to \rhd$

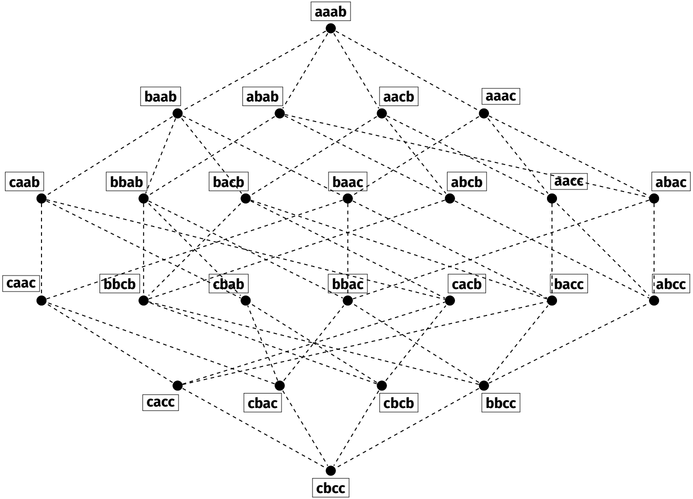


Figure: set of choice functions ordered wrt $\rhd$.

III. random choice model

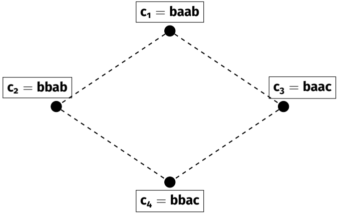
a random choice function (RCF) ρ assigns each choice set S a probability measure over S.

ρ	a	b	c
$\{a, b, c\}$	0	1	0
$\{a, b\}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\{a, c\}$	1	0	0
$\{b, c\}$	$\frac{2}{3}$	0	$\frac{1}{3}$

random choice model

a RCF p is representable as a prob. dist. over a set of deterministic choice functions (Birkhoff-von Neumann Thm).

ρ	a	b	c
$\{a,b,c\}$	0	1	0
$\{a,b\}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\{a,c\}$	1	0	0
$\{b,c\}$	$\frac{2}{3}$	0	$\frac{1}{3}$

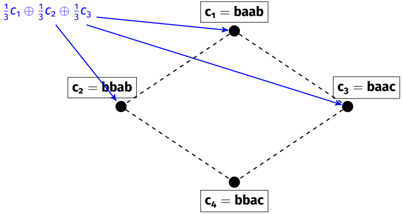


The **random choice model** $\Delta(\mu)$ associated with μ is the set of RCFs that are representable as a prob. dist. over choice functions in μ.

random choice model

a RCF p is representable as a prob. dist. over a set of deterministic choice functions (Birkhoff-von Neumann Thm).

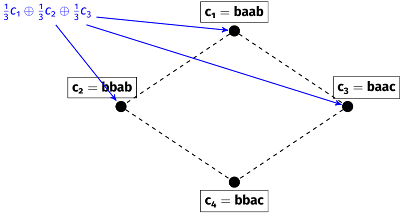
ρ	a	b	c
$\{a,b,c\}$	0	1	0
$\{a,b\}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\{a,c\}$	1	0	0
$\{b,c\}$	$\frac{2}{3}$	0	$\frac{1}{3}$



random choice model

a RCF p is representable as a prob. dist. over a set of deterministic choice functions (Birkhoff-von Neumann Thm).

ρ	a	b	c
$\{a,b,c\}$	0	1	0
$\{a,b\}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\{a,c\}$	1	0	0
$\{b,c\}$	$\frac{2}{3}$	0	$\frac{1}{3}$

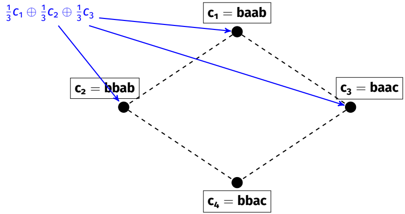


The **random choice model** $\Delta(\mu)$ associated with μ is the set of RCFs that are representable as a prob. dist. over choice functions in μ.

random choice model

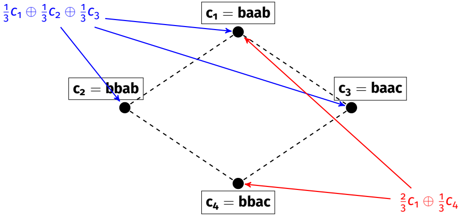
a RCF p is representable as a prob. dist. over a set of deterministic choice functions (Birkhoff-von Neumann Thm).

ρ	a	b	c
$\{a, b, c\}$	0	1	0
$\{a, b\}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\{a, c\}$	1	0	0
$\{b, c\}$	$\frac{2}{3}$	0	$\frac{1}{3}$



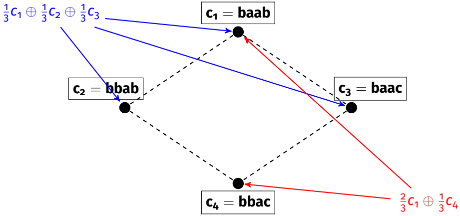
progressive (orderly) representation

ρ	a	b	c
$\{a, b, c\}$	0	1	0
$\{a, b\}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\{a, c\}$	1	0	0
$\{b, c\}$	0	$\frac{2}{3}$	$\frac{1}{3}$



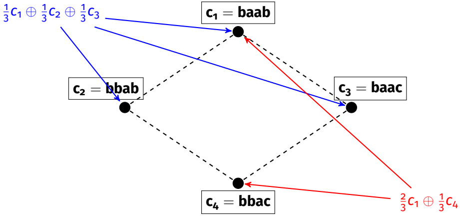
progressive (orderly) representation

ρ	a	b	c
$\{a,b,c\}$	0	1	0
$\{a,b\}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\{a,c\}$	1	0	0
$\{b,c\}$	0	$\frac{2}{3}$	$\frac{1}{3}$



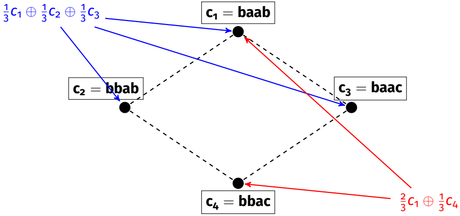
progressive (orderly) representation

ρ	a	b	c
$\{a,b,c\}$	0	1	0
$\{a,b\}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\{a,c\}$	1	0	0
$\{b,c\}$	0	$\frac{2}{3}$	$\frac{1}{3}$



progressive (orderly) representation

ρ	a	b	c
$\{a,b,c\}$	0	1	0
$\{a,b\}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\{a,c\}$	1	0	0
$\{b,c\}$	0	$\frac{2}{3}$	$\frac{1}{3}$

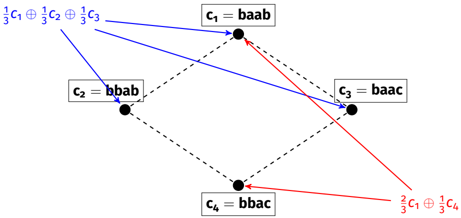


$\frac{2}{3}c_1 \oplus \frac{1}{3}c_4$ is a progressive representation since $c_1 \rhd c_4$

$\frac{1}{3}c_1 \oplus \frac{1}{3}c_2 \oplus \frac{1}{3}c_3$ is not since $c_2 \perp c_3$

progressive (orderly) representation

ρ	a	b	c
$\{a,b,c\}$	0	1	0
$\{a,b\}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\{a,c\}$	1	0	0
$\{b,c\}$	0	$\frac{2}{3}$	$\frac{1}{3}$



$\frac{2}{3}c_1 \oplus \frac{1}{3}c_4$ is a progressive representation since $c_1 \triangleright c_4$

$\frac{1}{3}c_1 \oplus \frac{1}{3}c_2 \oplus \frac{1}{3}c_3$ is not since $c_2 \perp c_3$

: a **self-progressive choice model** is a language that always provides for unique progressive representation.

self-progressiveness: 

$\Delta(\mu)$: random choice model obtained from a choice model μ

$\triangleright$: domination relation obtained from $>$ (**given & fixed**).

Defn: *A choice model μ is **self-progressive** wrt $\triangleright$ if each RCF $\rho \in \Delta(\mu)$ is uniquely representable as a prob. dist. <u>over elements of μ that are comparable to each other.</u>*

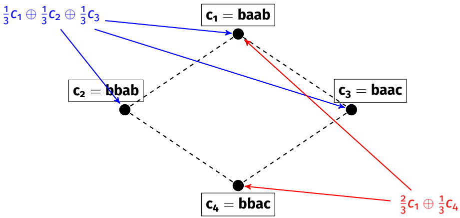
self-progressiveness: 

$\Delta(\mu)$: random choice model obtained from a choice model μ

$\triangleright$: domination relation obtained from $>$.

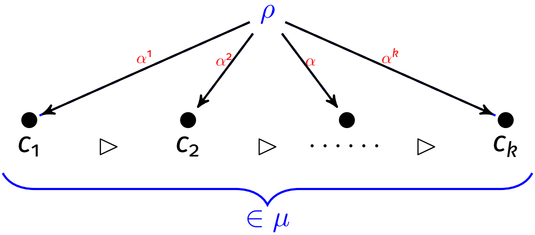
Defn: *A choice model μ is* ***self-progressive*** *wrt $\triangleright$ if each RCF $\rho \in \Delta(\mu)$ is uniquely representable as a prob. dist. over elements of μ that are comparable to each other.*

ρ	a	b	c
$\{a,b,c\}$	0	1	0
$\{a,b\}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\{a,c\}$	1	0	0
$\{b,c\}$	0	$\frac{2}{3}$	$\frac{1}{3}$



self-progressiveness 

Defn: *A choice model μ is **self-progressive** wrt $\triangleright$ if each RCF $\rho \in \Delta(\mu)$ is uniquely representable as a prob. distribution over a set of choice fncs. $\{c_i\}_{i=1}^{k} \subset \mu$ s.t. $c_1 \triangleright c_2 \cdots \triangleright c_k$.*



example: single-crossing RUM

- let $\mu = \{c_i\}_{i=1}^4$ be choice functions rationalized by $\{\succ_i\}_{i=1}^4$

$>$	$\succ_1$		$\succ_2$		$\succ_3$		$\succ_4$
a	a		b		b		c
b	b	$\rhd$	a	$\rhd$	c	$\rhd$	b
c	c		c		a		a

example: single-crossing RUM

- let $\mu = \{c_i\}_{i=1}^4$ be choice functions rationalized by $\{\succ_i\}_{i=1}^4$

$>$	$\succ_1$		$\succ_2$		$\succ_3$		$\succ_4$
a	a		b		b		c
b	b	$\triangleright$	a	$\triangleright$	c	$\triangleright$	b
c	c		c		a		a

- $\{\succ_i\}_{i=1}^k$ is single-crossing wrt $>$: $\forall\, x > y$
  if $x \succ_j y$, then $x \succ_i y$ for every i preceding j.

e.g. CRRA utilities parameterized by risk aversion coefficient.

example: single-crossing RUM

- let $\mu = \{c_i\}_{i=1}^4$ be choice functions rationalized by $\{\succ_i\}_{i=1}^4$

$>$	$\succ_1$		$\succ_2$		$\succ_3$		$\succ_4$
a	a		b		b		c
b	b	$\triangleright$	a	$\triangleright$	c	$\triangleright$	b
c	c		c		a		a

- $\{\succ_i\}_{i=1}^k$ is single-crossing wrt $>$: $\forall\, x > y$ if $x \succ_j y$, then $x \succ_i y$ for every i preceding j.

e.g. CRRA utilities parameterized by risk aversion coefficient.

- Apesteguia et al.'17: If a RCF is represented as a prob. dist. over comparable rational choice fncs. (SCRUM), then the representation is unique,

example: single-crossing RUM

- let $\mu = \{c_i\}_{i=1}^4$ be choice functions rationalized by $\{\succ_i\}_{i=1}^4$

$>$	$\succ_1$		$\succ_2$		$\succ_3$		$\succ_4$
a	a		b		b		c
b	b	$\triangleright$	a	$\triangleright$	c	$\triangleright$	b
c	c		c		a		a

- $\{\succ_i\}_{i=1}^k$ is single-crossing wrt $>$: $\forall\, x > y$
  if $x \succ_j y$, then $x \succ_i y$ for every i preceding j.

e.g. CRRA utilities parameterized by risk aversion coefficient.

- Apesteguia et al.'17: If a RCF is represented as a prob. dist. over comparable rational choice fncs. (SCRUM), then the representation is unique, **i.e. SCRUM is self-progressive**.

connection to the literature

- Apestaguia et al.'17: If a RCF is represented as a prob. dist. over a set of comparable rational choice fncs, then the representation is unique, **i.e. SCRUM is self-progressive.**

e.g. CRRA utilities parameterized by risk aversion coefficient.

- However, parametrizing choices according to multiple behavioral characteristics is critical in explaining economic phenomena.

connection to the literature

- Apestaguia et al.'17: If a RCF is represented as a prob. dist. over a set of comparable rational choice fncs, then the representation is unique, **i.e. SCRUM is self-progressive.**

e.g. CRRA utilities parameterized by risk aversion coefficient.

- However, parametrizing choices according to multiple behavioral characteristics is critical in explaining economic phenomena.

e.g. The "equity premium puzzle"

  - Epstein & Zin'89: risk aversion & elasticity of substitution
  - Benartzi & Thaler'95: loss aversion & frequent evaluations

connection to the literature

- Apesteguia et al.'17: If a RCF is represented as a prob. dist. over a set of comparable rational choice fncs, then the representation is unique, **i.e. SCRUM is self-progressive.**
- Filiz-Ozbay & Masatlioglu'22: a RCF is uniquely representable as a prob. dist. over comparable choice fncs,

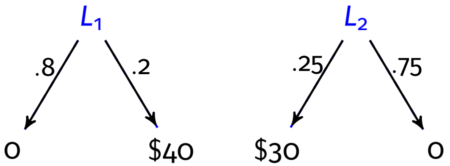
connection to the literature

- Apesteguia et al.'17: If a RCF is represented as a prob. dist. over a set of comparable rational choice fncs, then the representation is unique, **i.e. SCRUM is self-progressive.**
- Filiz-Ozbay & Masatlioglu'22: a RCF is uniquely representable as a prob. dist. over comparable choice fncs, **i.e.** $\mu = \{$**all choice functions**$\}$ **is self-progressive.**

literature $\rightarrow$ *existence of unique progressive representation*

Ex 2: similarity-based choice (Rubinstein'88)

- to choose (m_1, p_1) or (m_2, p_2), agent i first checks if "p_1 is similar to p_2 & m_1 is different from m_2", or vice versa.



- If one of these is true, then the differentiating dimension becomes decisive.Otherwise, i chooses the $>$-better one.

agent *i* is described by (ϵ^i, δ^i) with $\delta^i \geq \epsilon^i$:

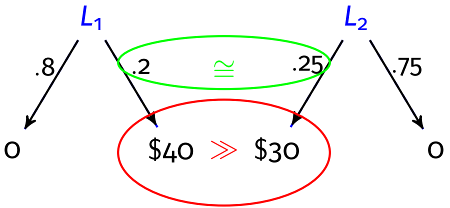
$\cong$: "t_1 is similar to t_2" if $|t_1 - t_2| < \epsilon^i$

$\gg$: "t_1 is different from t_2" if $|t_1 - t_2| > \delta^i$

Q: Is this model self-progressive?

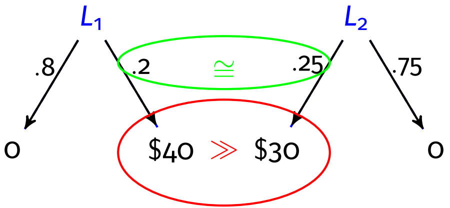
Ex 2: similarity-based choice (Rubinstein'88)

- to choose (m_1, p_1) or (m_2, p_2), agent i first checks if "p_1 is similar to p_2 & m_1 is different from m_2", or vice versa.



Ex 2: similarity-based choice (Rubinstein'88)

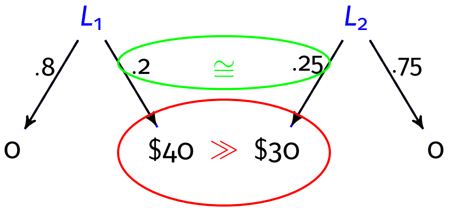
- to choose (m_1, p_1) or (m_2, p_2), agent i first checks if "p_1 is similar to p_2 & m_1 is different from m_2", or vice versa.



- If one of these is true, then the differentiating dimension becomes decisive.

Ex 2: similarity-based choice (Rubinstein'88)

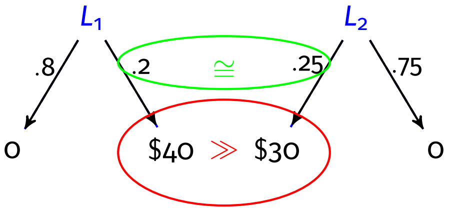
- to choose (m_1, p_1) or (m_2, p_2), agent i first checks if "p_1 is similar to p_2 & m_1 is different from m_2", or vice versa.



- If one of these is true, then the differentiating dimension becomes decisive.Otherwise, i chooses the $>$-better one.

Ex 2: similarity-based choice (Rubinstein'88)

- to choose (m_1, p_1) or (m_2, p_2), agent i first checks if "p_1 is similar to p_2 & m_1 is different from m_2", or vice versa.



- If one of these is true, then the differentiating dimension becomes decisive.Otherwise, i chooses the $>$-better one.

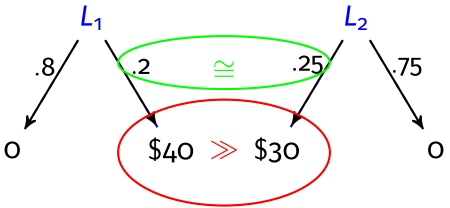
agent i is described by (ϵ^i, δ^i) with $\delta^i \geq \epsilon^i$:

$\cong$: "t_1 is similar to t_2" if $|t_1 - t_2| < \epsilon^i$

$\gg$: "t_1 is different from t_2" if $|t_1 - t_2| > \delta^i$

Ex 2: similarity-based choice (Rubinstein'88)

- to choose (m_1, p_1) or (m_2, p_2), agent i first checks if "p_1 is similar to p_2 & m_1 is different from m_2", or vice versa.



- If one of these is true, then the differentiating dimension becomes decisive.Otherwise, i chooses the $>$-better one.

agent *i* is described by (ϵ^i, δ^i) with $\delta^i \geq \epsilon^i$:

$\cong$: "t_1 is similar to t_2" if $|t_1 - t_2| < \epsilon^i$

$\gg$: "t_1 is different from t_2" if $|t_1 - t_2| > \delta^i$

Q: Is this model self-progressive?

questions

- Which choice models are self-progressive?
  Hope: A simple test?

questions

- Which choice models are self-progressive?
  Hope: A simple test?

- Is there a simple procedure to obtain the progressive representation (within a given model)?

questions

- Which choice models are self-progressive?
  Hope: A simple test?

- Is there a simple procedure to obtain the progressive representation (within a given model)?

- Can we obtain a “recipe” for self-progressiveness?

an answer

Thm 1: *A choice model μ is self-progressive wrt $\rhd$ iff the pair $\langle \mu, \rhd \rangle$ is a lattice.*

lattice?

Defn: $\langle \mu, \rhd \rangle$ *is a lattice if for each* $c, c' \in \mu$*, we have their join* $c \vee c'$ *and meet* $c \wedge c'$ *are in* μ *as well.*

lattice?

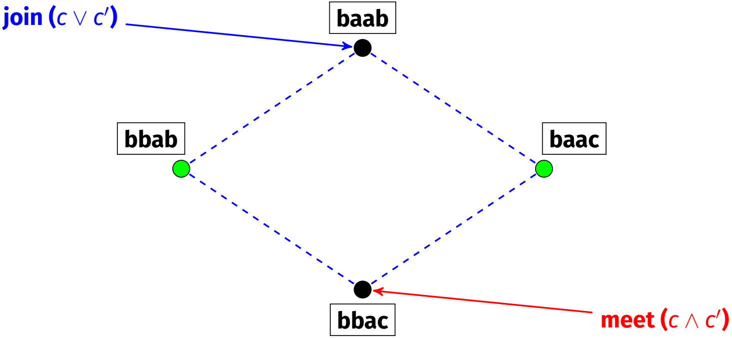
Defn: *$\langle \mu, \rhd \rangle$ is a lattice if for each $c, c' \in \mu$, we have their join $c \vee c'$ and meet $c \wedge c'$ are in μ as well.*

For each pair of choice fncs. c and c', their

- **join:** $c \vee c'(S) = max(\{c(S), c'(S)\}, >)$
- **meet:** $c \wedge c'(S) = min(\{c(S), c'(S)\}, >)$

for each choice set S.

Defn: *$\langle \mu, \triangleright \rangle$ is a lattice if for each $c, c' \in \mu$, we have their join $c \vee c'$ and meet $c \wedge c'$ are in μ as well.*



primitive ordering: $a > b > c \to \rhd$

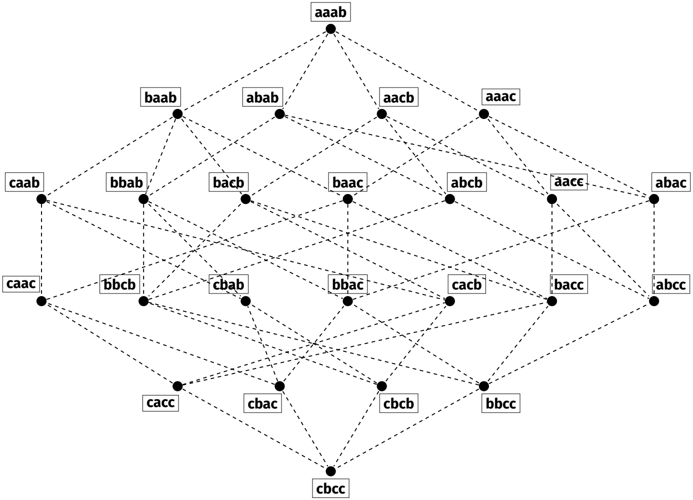


Figure: choice functions ordered wrt $\rhd$.

primitive ordering: $a > b > c \rightarrow \rhd$

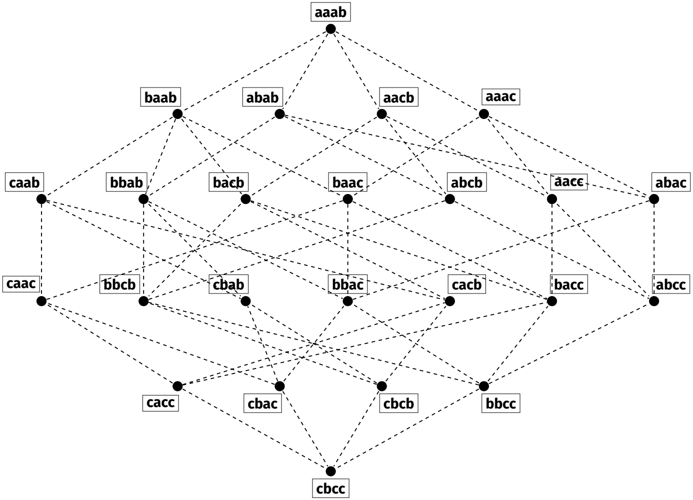


Figure: choice functions lattice wrt $\rhd$.

primitive ordering: $a > b > c \rightarrow \; \triangleright$

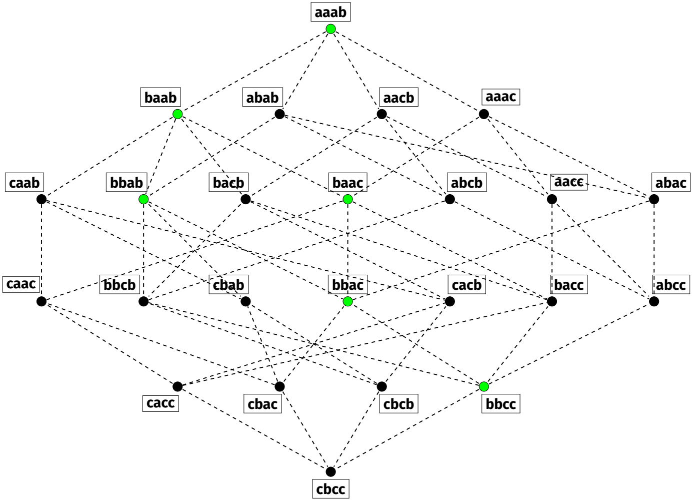
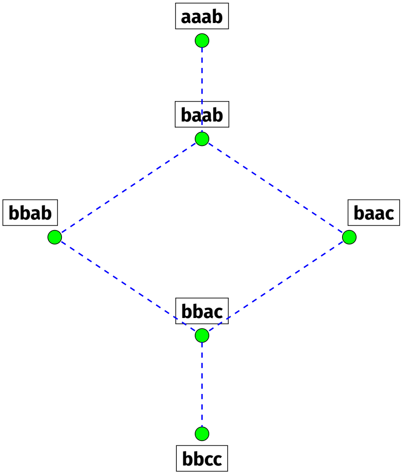
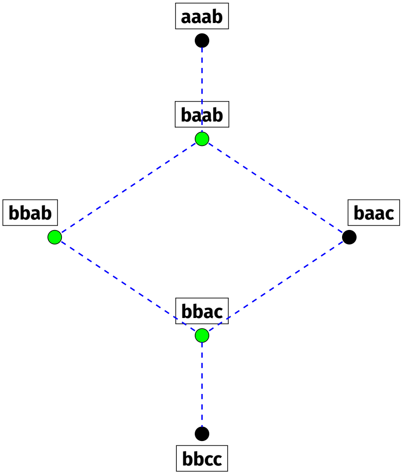


Figure: choice functions lattice wrt $\triangleright$.

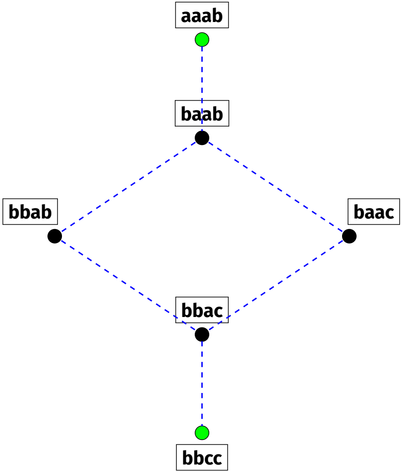
Defn: *$\langle \mu, \rhd \rangle$ is a lattice if for each $c, c' \in \mu$, we have their join $c \vee c'$ and meet $c \wedge c'$ are in μ as well.*



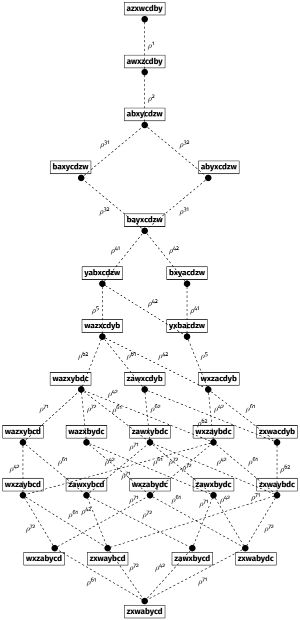
Defn: *$\langle \mu, \triangleright \rangle$ is a lattice if for each $c, c' \in \mu$, we have their join $c \vee c'$ and meet $c \wedge c'$ are in μ as well.*



Defn: *$\langle \mu, \rhd \rangle$ is a lattice if for each $c, c' \in \mu$, we have their join $c \vee c'$ and meet $c \wedge c'$ are in μ as well.*



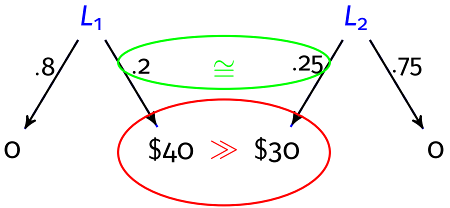
can be scary!



azxwcdby
awxzcdby
abxycdzw
baxycdzw
abyxcdzw
bayxcdzw
yabxcdzw
bxyacdzw
wazxcdyb
yxbacdzw
wazxybdc
zawxcdyb
wxzacdyb
wazxybcd
wazxbydc
zawxybdc
wxzaybdc
zxwacdyb
wxzaybcd
zawxybcd
wxzabydc
zawxbydc
zxwaybdc
wxzabycd
zxwaybcd
zawxbycd
zxwabydc
zxwabycd

Ex 2: similarity-based choice (Rubinstein'88)

- to choose (m_1, p_1) or (m_2, p_2), agent i first checks if "p_1 is similar to p_2 & m_1 is different from m_2", or vice versa.



- If one of these is true, then the differentiating dimension becomes decisive. Otherwise, i chooses the $>$-better one.

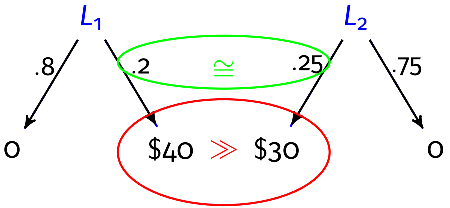
agent *i* is described by (ϵ^i, δ^i) with $\delta^i \geq \epsilon^i$:

$\cong$: "t_1 is similar to t_2" if $|t_1 - t_2| < \epsilon^i$

$\gg$: "t_1 is different from t_2" if $|t_1 - t_2| > \delta^i$

Ex 2: similarity-based choice (Rubinstein'88)

- to choose (m_1, p_1) or (m_2, p_2), agent i first checks if "p_1 is similar to p_2 & m_1 is different from m_2", or vice versa.



- If one of these is true, then the differentiating dimension becomes decisive. Otherwise, i chooses the $>$-better one.

agent i is described by (ϵ^i, δ^i) with $\delta^i \geq \epsilon^i$:

$\cong$: "t_1 is similar to t_2" if $|t_1 - t_2| < \epsilon^i$

$\gg$: "t_1 is different from t_2" if $|t_1 - t_2| > \delta^i$

$c^i \vee c^j$ can be described by $(min(\epsilon^i, \epsilon^j), max(\delta^i, \delta^j))$

$c^i \wedge c^j$ can be described by $(max(\epsilon^i, \epsilon^j), min(\delta^i, \delta^j))$

proof sketch

Thm 1: *A choice model μ is self-progressive wrt $\triangleright$ iff the pair $\langle \mu, \triangleright \rangle$ is a lattice.*

proof sketch

Thm 1: *A choice model μ is self-progressive wrt $\rhd$ iff the pair $\langle \mu, \rhd \rangle$ is a lattice.*

Only if: Let $c, c' \in \mu$ and $\rho = \frac{1}{2}c \oplus \frac{1}{2}c'$.

Unique progressive representation: $\frac{1}{2}(c \vee c') \oplus \frac{1}{2}(c \wedge c')$.

Since μ is self-progressive, $c \vee c' \in \mu$ and $c \wedge c' \in \mu$.

proof of if part

We will decompose each $\rho \in \Delta(\mu)$ into a set of comparable choice fncs in μ, by using a probabilistic procedure.

proof of if part

We will decompose each $\rho \in \Delta(\mu)$ into a set of comparable choice fncs in μ, by using a probabilistic procedure.

If part: uniform decomposition procedure

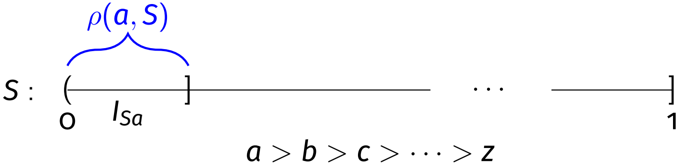
Suppose that $\langle \mu, \rhd \rangle$ is a lattice, and let $\rho \in \Delta(\mu)$.

Step 1: For each S, partition $(0, 1]$ interval into half open intervals $I_{Sx} = (l_{Sx}, u_{Sx}]$ with length $\rho(x, S)$, descending in $>$.

If part: uniform decomposition procedure

Suppose that $\langle \mu, \rhd \rangle$ is a lattice, and let $\rho \in \Delta(\mu)$.

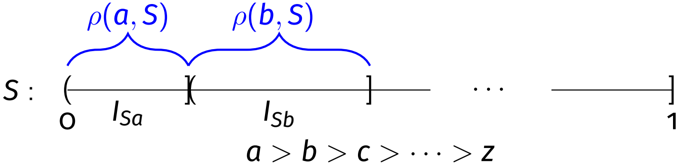
Step 1: For each S, partition $(0, 1]$ interval into half open intervals $I_{Sx} = (l_{Sx}, u_{Sx}]$ with length $\rho(x, S)$, descending in $>$.



If part: uniform decomposition procedure

Suppose that $\langle \mu, \triangleright \rangle$ is a lattice, and let $\rho \in \Delta(\mu)$.

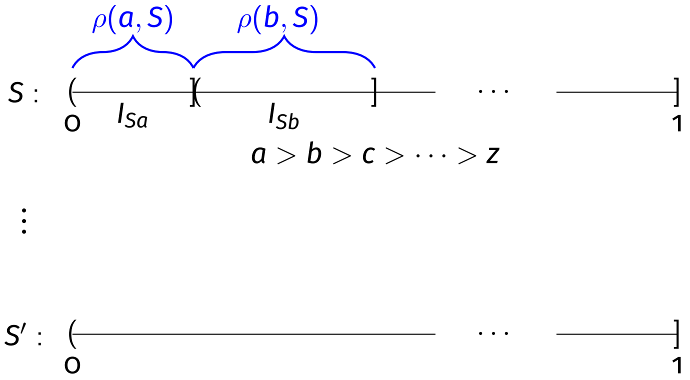
Step 1: For each S, partition $(0, 1]$ interval into half open intervals $I_{Sx} = (l_{Sx}, u_{Sx}]$ with length $\rho(x, S)$, descending in $>$.



If part: uniform decomposition procedure

Suppose that $\langle \mu, \rhd \rangle$ is a lattice, and let $\rho \in \Delta(\mu)$.

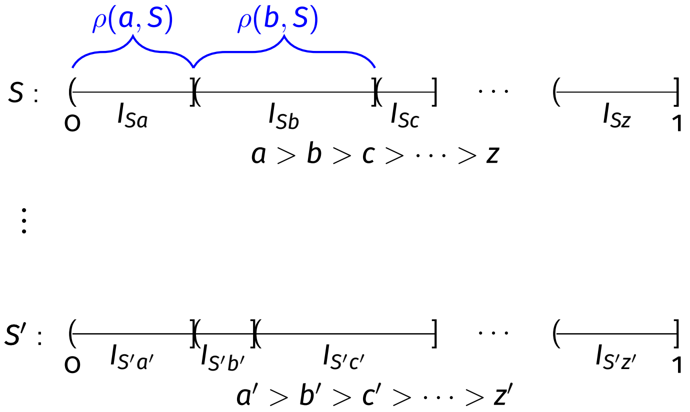
Step 1: For each S, partition $(0, 1]$ interval into half open intervals $I_{Sx} = (l_{Sx}, u_{Sx}]$ with length $\rho(x, S)$, descending in $>$.



If part: uniform decomposition procedure

Suppose that $\langle \mu, \triangleright \rangle$ is a lattice, and let $\rho \in \Delta(\mu)$.

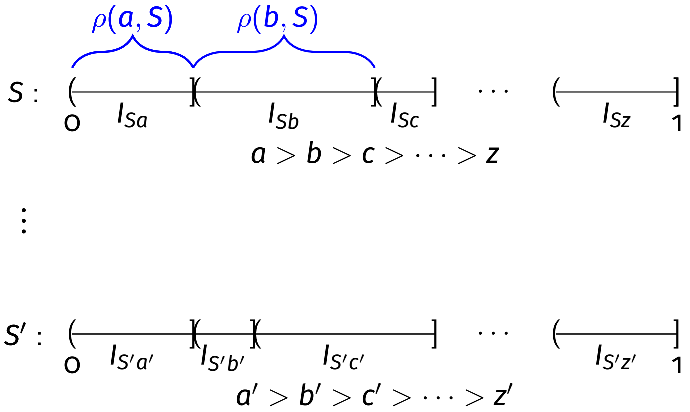
Step 1: For each S, partition $(0, 1]$ interval into half open intervals $I_{Sx} = (l_{Sx}, u_{Sx}]$ with length $\rho(x, S)$, descending in $>$.



If part: uniform decomposition procedure

Suppose that $\langle \mu, \rhd \rangle$ is a lattice, and let $\rho \in \Delta(\mu)$.

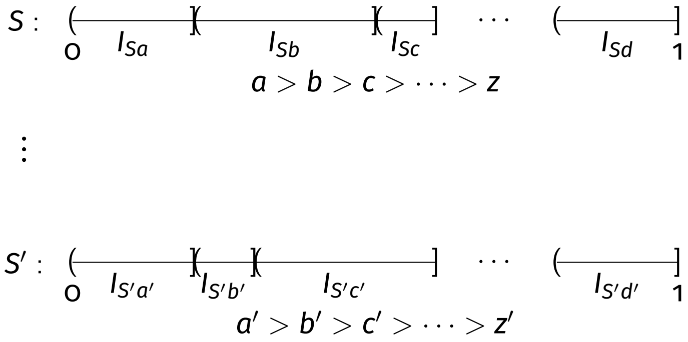
Step 1: For each S, partition $(0, 1]$ interval into half open intervals $I_{Sx} = (l_{Sx}, u_{Sx}]$ with length $\rho(x, S)$, descending in $>$.



If part: uniform decomposition procedure

Suppose that $\langle \mu, \rhd \rangle$ is a lattice, and let $\rho \in \Delta(\mu)$.

Step 2: Pick a real number $r \sim \mathbb{U}(0,1]$, and for each (S, x) let $c(S) = x$ iff $r \in I_{Sx}$.

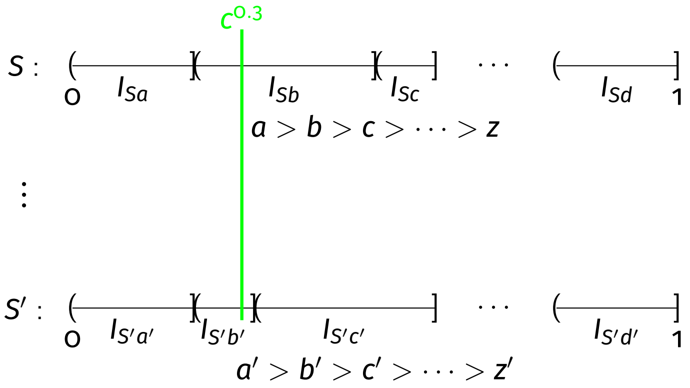


If $\mu = \{$all choice fncs.$\}$, then we get Thm 1 of F-O&M'22.

If part: uniform decomposition procedure

Suppose that $\langle \mu, \rhd \rangle$ is a lattice, and let $\rho \in \Delta(\mu)$.

Step 2: Pick a real number $r \sim \mathbb{U}(0,1]$, and for each (S, x) let $c(S) = x$ iff $r \in I_{Sx}$.

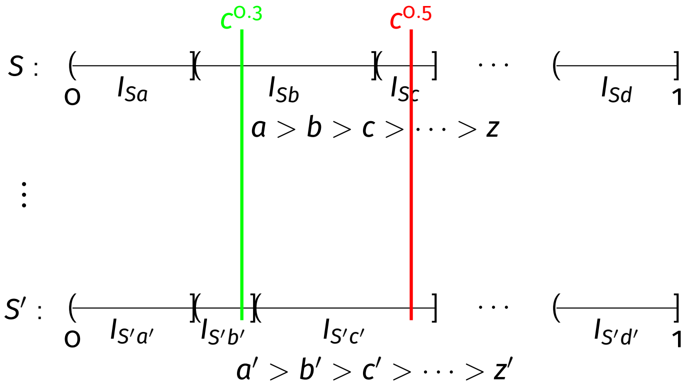


If $\mu = \{$all choice fncs.$\}$, then we get Thm 1 of F-O&M'22.

If part: uniform decomposition procedure

Suppose that $\langle \mu, \rhd \rangle$ is a lattice, and let $\rho \in \Delta(\mu)$.

Step 2: Pick a real number $r \sim \mathbb{U}(0, 1]$, and for each (S, x) let $c(S) = x$ iff $r \in I_{Sx}$.

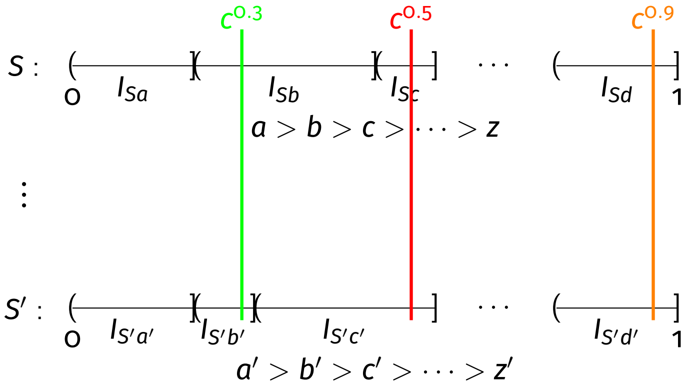


If $\mu = \{$all choice fncs.$\}$, then we get Thm 1 of F-O&M'22.

If part: uniform decomposition procedure

Suppose that $\langle \mu, \triangleright \rangle$ is a lattice, and let $\rho \in \Delta(\mu)$.

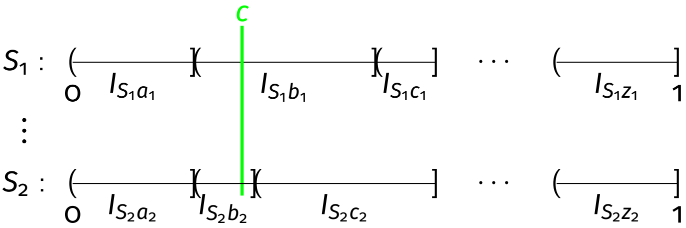
Step 2: Pick a real number $r \sim \mathbb{U}(0,1]$, and for each (S, x) let $c(S) = x$ iff $r \in I_{Sx}$.



If $\mu = \{$all choice fncs.$\}$, then we get Thm 1 of F-O&M'22.

part I

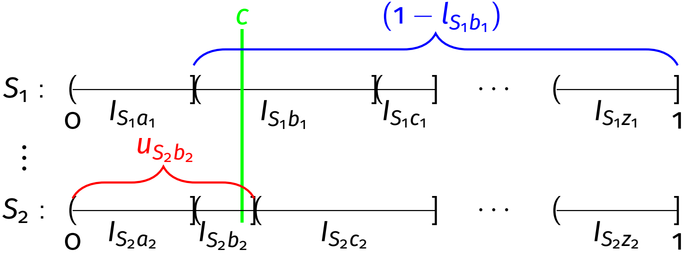
- let c be a choice fnc. found by UDP.
- fix S_1 & S_2; let $b_1 = c(S_1)$ & $b_2 = c(S_2)$.



WTS: $\exists\, c_{12} \in \mu$ s.t. $c_{12}(S_1) = b_1$ and $c_{12}(S_2) = b_2$.

WTS: $\exists\, c_{12} \in \mu$ s.t. $c_{12}(S_1) = b_1$ and $c_{12}(S_2) = b_2$.

- note that $(1 - l_{S_1 b_1}) + u_{S_2 b_2} > 1$:

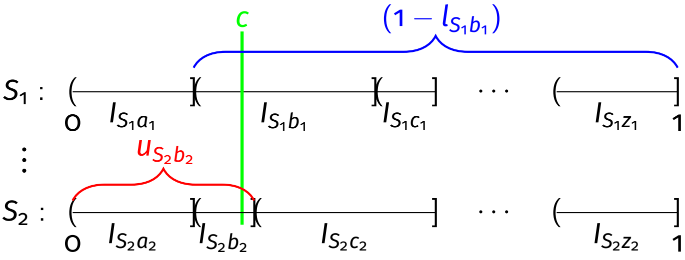


$\Rightarrow\ \exists$

	S_1	S_2
$c_1 \in \mu$:	$\leq b_1$	$\geq b_2$

WTS: $\exists\, c_{12} \in \mu$ s.t. $c_{12}(S_1) = b_1$ and $c_{12}(S_2) = b_2$.

- note that $(1 - l_{S_1 b_1}) + u_{S_2 b_2} > 1$:

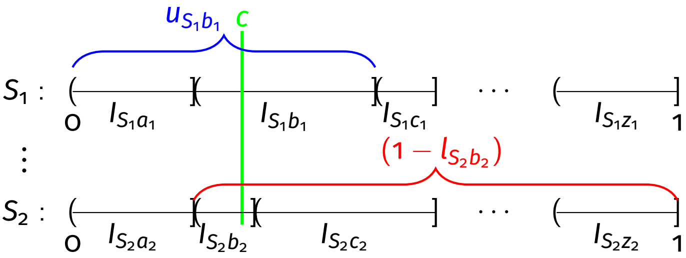


$\Rightarrow\ \exists$

	S_1	S_2
$c_1 \in \mu$:	$= b_1$	$\geq b_2$

WTS: $\exists\, c_{12} \in \mu$ s.t. $c_{12}(S_1) = b_1$ and $c_{12}(S_2) = b_2$.

- note that $u_{S_1 b_1} + (1 - l_{S_2 b_2}) > 1$:

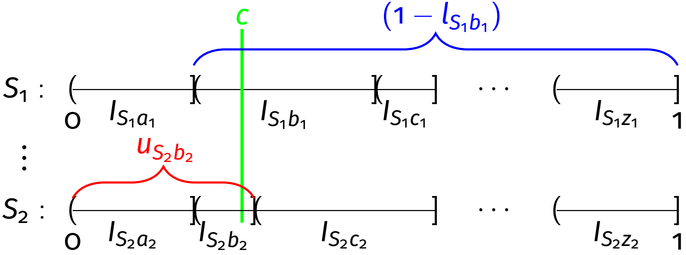


$\Rightarrow\ \exists$

	S_1	S_2
$c_1 \in \mu$:	$= b_1$	$\geq b_2$
$c_2 \in \mu$:	$\geq b_1$	$= b_2$

WTS: $\exists\, c_{12} \in \mu$ s.t. $c_{12}(S_1) = b_1$ and $c_{12}(S_2) = b_2$.

- note that $(1 - l_{S_1 b_1}) + u_{S_2 b_2} > 1$:



$\Rightarrow\ \exists$

	S_1	S_2
$c_1 \in \mu$:	$\geq b_1$	$= b_2$
$c_2 \in \mu$:	$= b_1$	$\geq b_2$
$c_{12} = c_1 \wedge c_2$:	$= b_1$	$= b_2$

part II

(*) for each pair of choice sets S_1 & S_2, there exists $c_{12} \in \mu$ that copies c on S_1 & S_2.

Extension lemma: Let $\langle \mu, \rhd \rangle$ be a lattice.

part II

(*) for each pair of choice sets S_1 & S_2, there exists $c_{12} \in \mu$ that copies c on S_1 & S_2.

Extension lemma: Let $\langle \mu, \triangleright \rangle$ be a lattice. For any choice fnc. c, if (*) holds,

part II

(*) for each pair of choice sets S_1 & S_2, there exists $c_{12} \in \mu$ that copies c on S_1 & S_2.

Extension lemma: Let $\langle \mu, \triangleright \rangle$ be a lattice. For any choice fnc. c, if (*) holds, then $c \in \mu$.

Proof: Consider any $S_1, S_2, S_3 \in \Omega$.

WTS: $\exists\, c_{123} \in \mu$ s.t. $c_{123}(S_k) = c(S_k)$ for $k \in \{1, 2, 3\}$:

part II

(*) for each pair of choice sets S_1 & S_2, there exists $c_{12} \in \mu$ that copies c on S_1 & S_2.

Extension lemma: Let $\langle \mu, \triangleright \rangle$ be a lattice. For any choice fnc. c, if (*) holds, then $c \in \mu$.

Proof: Consider any $S_1, S_2, S_3 \in \Omega$.

WTS: $\exists\, c_{123} \in \mu$ s.t. $c_{123}(S_k) = c(S_k)$ for $k \in \{1, 2, 3\}$:

- $c_{123} \stackrel{\text{def}}{=} (c_{12} \wedge c_{13}) \vee (c_{12} \wedge c_{23}) \vee (c_{13} \wedge c_{23})$

part II

(*) for each pair of choice sets S_1 & S_2, there exists $c_{12} \in \mu$ that copies c on S_1 & S_2.

Extension lemma: Let $\langle \mu, \triangleright \rangle$ be a lattice. For any choice fnc. c, if (*) holds, then $c \in \mu$.

Proof: Consider any $S_1, S_2, S_3 \in \Omega$.

WTS: $\exists\, c_{123} \in \mu$ s.t. $c_{123}(S_k) = c(S_k)$ for $k \in \{1, 2, 3\}$:

- $c_{123} \stackrel{\text{def}}{=} (c_{12} \wedge c_{13}) \vee (c_{12} \wedge c_{23}) \vee (c_{13} \wedge c_{23}) \in \mu$

$c_{23}(S_1) = y$	$(c_{12} \wedge c_{13})$	$(c_{12} \wedge c_{23})$	$(c_{13} \wedge c_{23})$
$y > x$	x	x	x
$x \geq y$	x	y	y

QED

implications of Thm 1

i. We have a test for self-progressiveness.

implications of Thm 1

i. We have a test for self-progressiveness.

ii. We obtain a precise recipe to restrict or extend any choice model as to be self-progressive

implications of Thm 1

i. We have a test for self-progressiveness.

ii. We obtain a precise recipe to restrict or extend any choice model as to be self-progressive $\rightarrow$ minimal self-progressive extension of rational choice.

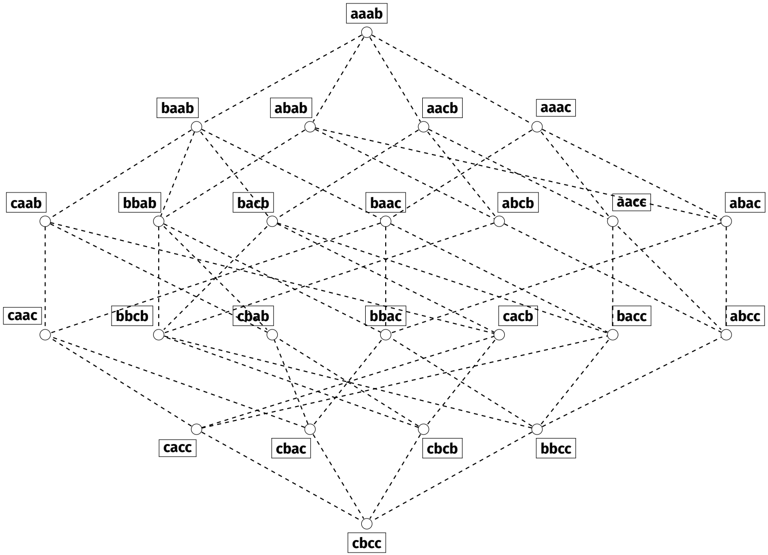
iii. We learn that self-progressive models allow for specifying multiple behavioral characteristics $\rightarrow$ examples.

Ex: rational choice

i. a test for self-progressiveness.

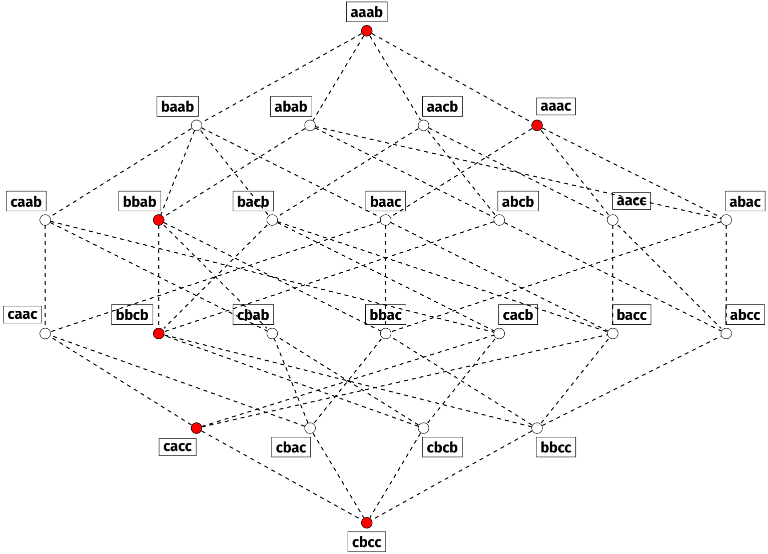
Ex: rational choice

i. a test for self-progressiveness.



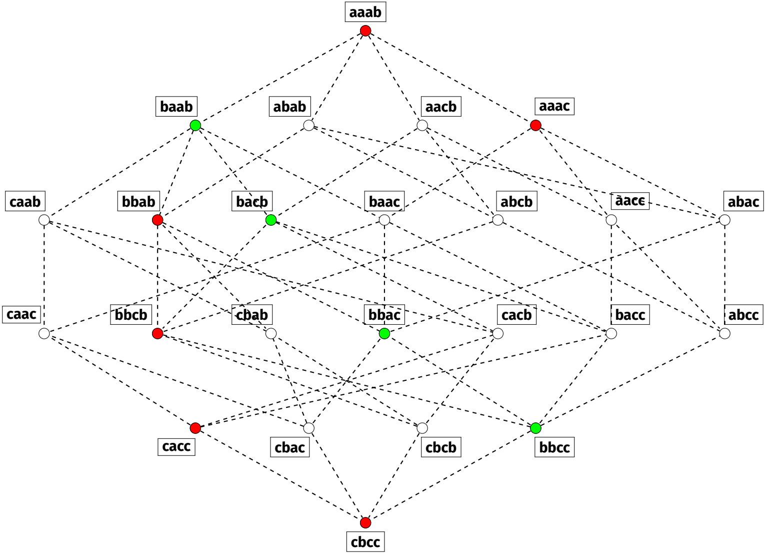
Ex: rational choice

i. a test for self-progressiveness.



Ex: rational choice

i. a test for self-progressiveness.

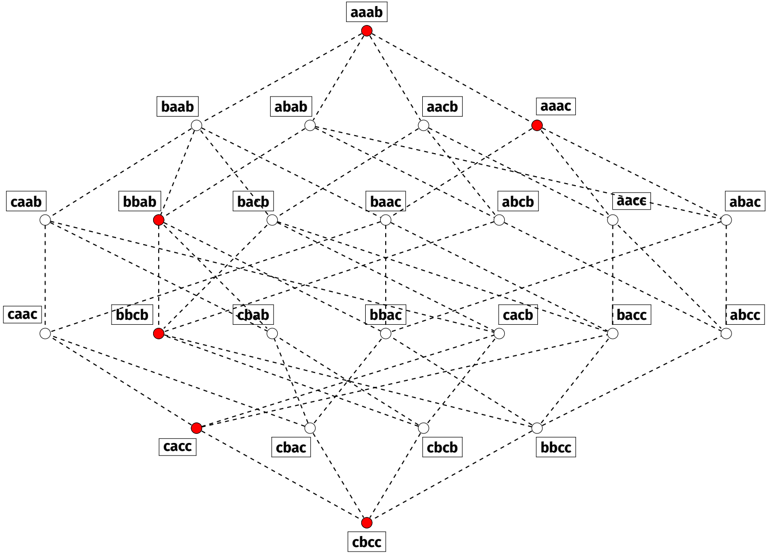


Ex: rational choice

ii. How to restrict for self-progressiveness?

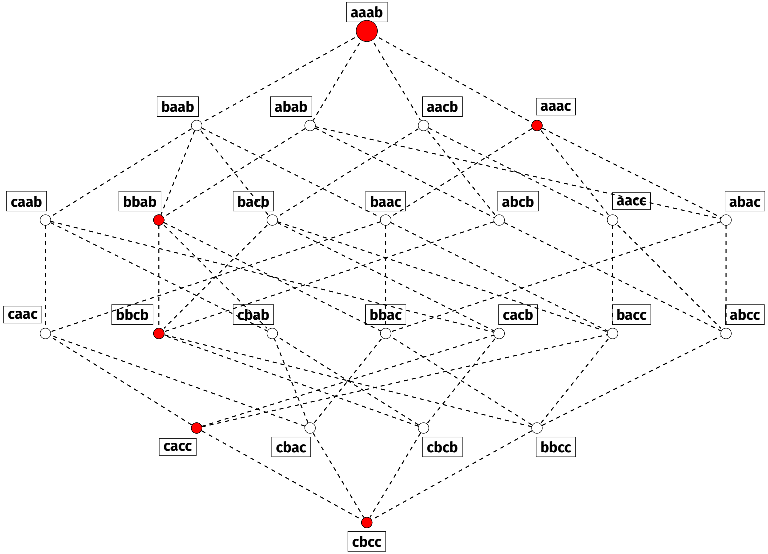
Ex: rational choice

ii. How to restrict for self-progressiveness?



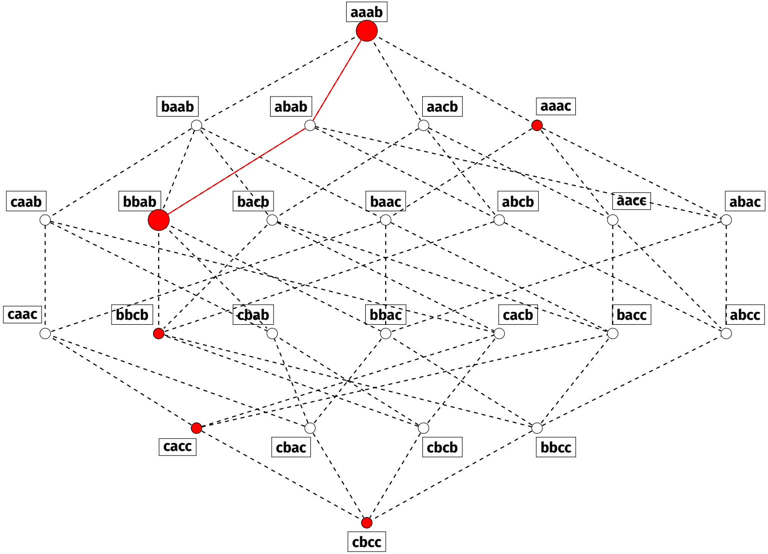
Ex: rational choice

ii. How to restrict for self-progressiveness?



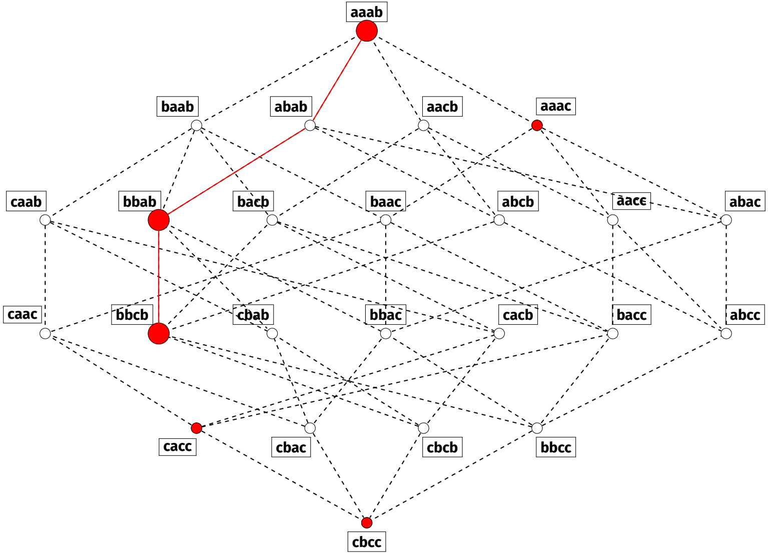
Ex: rational choice

ii. How to restrict for self-progressiveness?



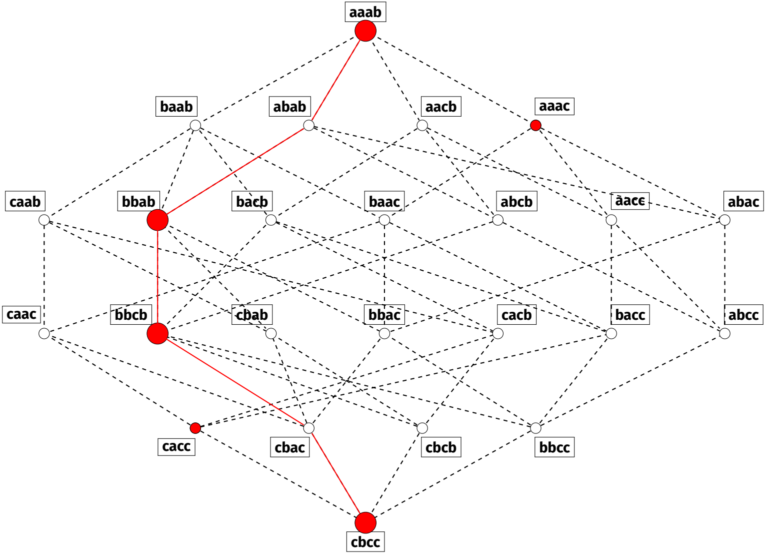
Ex: rational choice

ii. How to restrict for self-progressiveness?



Ex: rational choice

ii. How to restrict for self-progressiveness?



minimal self-prog. extension of rationals

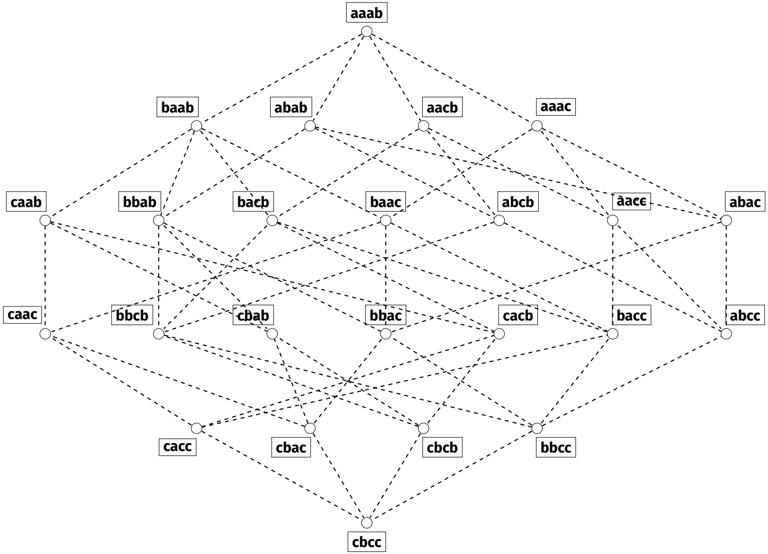
iii. How to extend for self-progressiveness?

minimal self-prog. extension of rationals

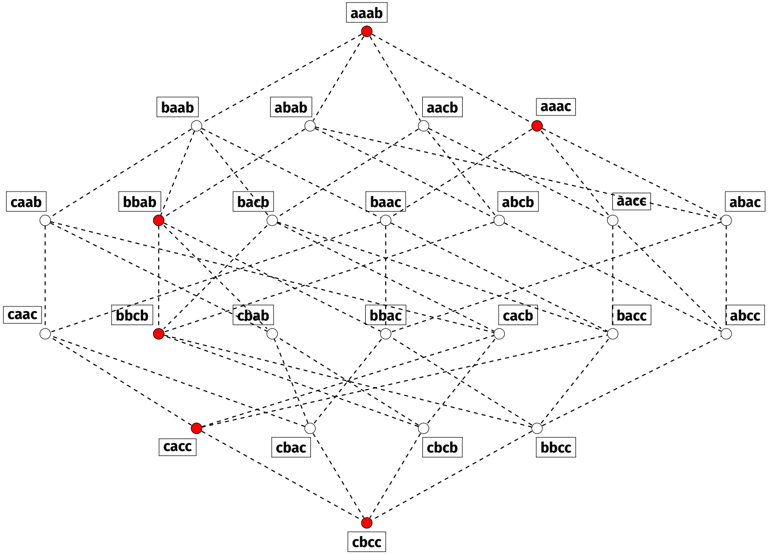
iii. How to extend for self-progressiveness?

- the extension is minimal if we are parsimonious in adding nonrational choice functions so that there is no self-progressive choice model
  - that contains rational choice functions, and
  - is contained in the minimal extension.

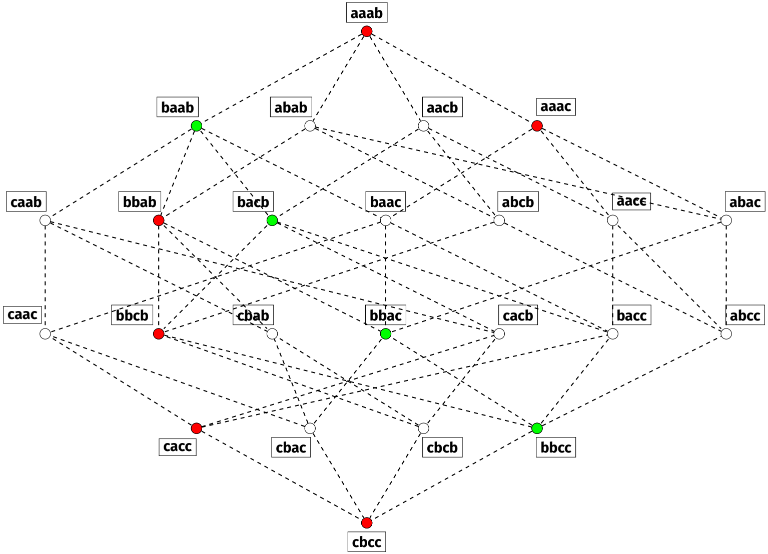
minimal self-prog. extension of rationals



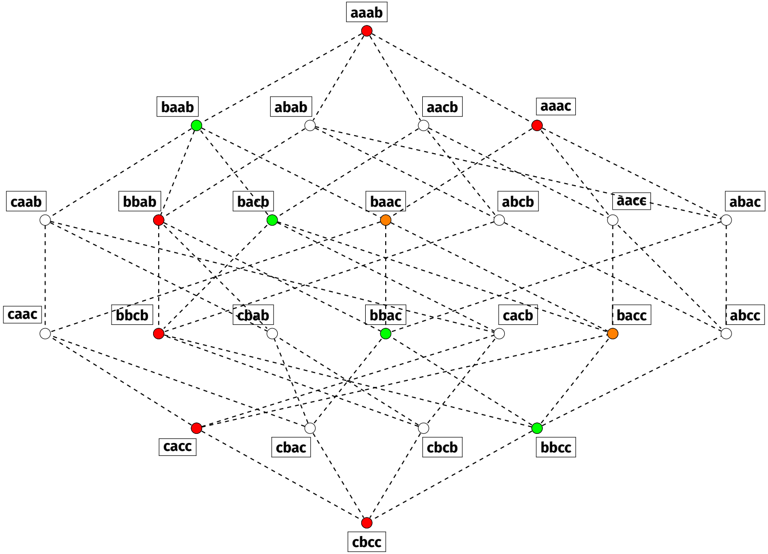
minimal self-prog. extension of rationals



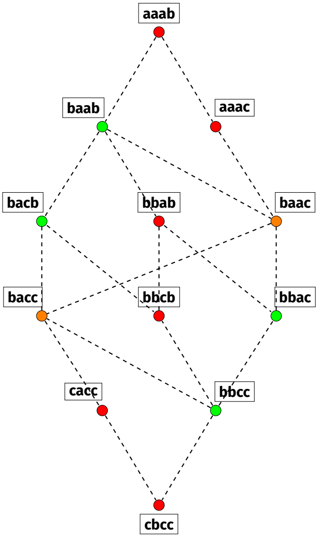
minimal self-prog. extension of rationals



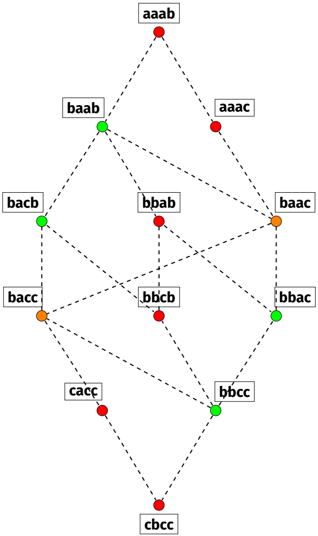
minimal self-prog. extension of rationals



the lattice extension of rational choice



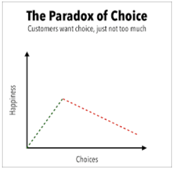
the lattice extension of rational choice



Question: Is there an underlying economic pattern?

Yes: a model of choice overload

choice overload: larger choice sets might make agents worse-off.



Yes: a model of choice overload

choice overload: larger choice sets might make agents worse-off.

A choice fnc $c \in \mu^\theta$

Yes: a model of choice overload

choice overload: larger choice sets might make agents worse-off.

A choice fnc $c \in \mu^{\theta}$ if for each choice set S, the chosen alternative gets $>$-better whenever we

Yes: a model of choice overload

choice overload: larger choice sets might make agents worse-off.

A choice fnc $c \in \mu^\theta$ if for each choice set S, the chosen alternative gets $>$-better whenever we

A1. remove alternatives that are worse than $c(S)$, or

Yes: a model of choice overload

choice overload: larger choice sets might make agents worse-off.

A choice fnc $c \in \mu^\theta$ if for each choice set S, the chosen alternative gets $>$-better whenever we

A1. remove alternatives that are worse than $c(S)$, or

A2. add alternatives that are better than $c(S)$.

a model of choice overload

A choice function $c \in \mu^\theta$ whenever for each S and $x \in S$,

A1. if $c(S) > x$ then $c(S \setminus x) \geq c(S)$, and

A2. if $x > c(S)$ then $c(S) \geq c(S \setminus x)$.

a model of choice overload

A choice function $c \in \mu^\theta$ whenever for each S and $x \in S$,

A1. if $c(S) > x$ then $c(S \setminus x) \geq c(S)$, and

A2. if $x > c(S)$ then $c(S) \geq c(S \setminus x)$.

(experimental findings by Chernev & Hamilton'09 are supportive)

a model of choice overload

A choice function $c \in \mu^\theta$ whenever for each S and $x \in S$,

A1. if $c(S) > x$ then $c(S \setminus x) \geq c(S)$, and

A2. if $x > c(S)$ then $c(S) \geq c(S \setminus x)$.

(experimental findings by Chernev & Hamilton'09 are supportive)

Thm 2: *μ^θ is the minimal self-progressive extension of rational choice model.*

identification of $>$

- Let μ be a given choice model and $x, y, z \in X$ be a triple.

identification of $>$

- Let μ be a given choice model and $x, y, z \in X$ be a triple.
- Then, y is **revealed to be between** x and z ($y \ \mathcal{B}_{\mu} \ \{x, z\}$)

identification of $>$

- Let μ be a given choice model and $x, y, z \in X$ be a triple.
- Then, y is **revealed to be between** x and z ($y \ \mathcal{B}_\mu \ \{x, z\}$) if $\exists c \in \mu$ s.t. $c(S) = y$ and $c(S \setminus z) = x$ for some $S \in \Omega$.

identification of $>$

- Let μ be a given choice model and $x, y, z \in X$ be a triple.
- Then, y is **revealed to be between** x and z ($y \ \mathcal{B}_\mu \ \{x, z\}$) if $\exists c \in \mu$ s.t. $c(S) = y$ and $c(S \setminus z) = x$ for some $S \in \Omega$.
- If $\mu \subseteq \mu^\theta(>)$, then $y \ \mathcal{B}_\mu \ \{x, z\} \Rightarrow x > y > z$ or $z < y < x$.

identification of $>$

- Let μ be a given choice model and $x, y, z \in X$ be a triple.
- Then, y is **revealed to be between** x and z ($y \; \mathcal{B}_\mu \; \{x, z\}$) if $\exists c \in \mu$ s.t. $c(S) = y$ and $c(S \setminus z) = x$ for some $S \in \Omega$.
- If $\mu \subseteq \mu^\theta(>)$, then $y \; \mathcal{B}_\mu \; \{x, z\} \Rightarrow x > y > z$ or $z < y < x$.

identification of $>$

x is **revealed to be between** y and z ($y \mathcal{B}_\mu \{x, z\}$)
if $\exists c \in \mu$ s.t. $c(S) = y$ and $c(S \setminus z) = x$ for some $S \in \Omega$.

identification of $>$

x is **revealed to be between** y and z ($y \, \mathcal{B}_\mu \, \{x, z\}$)
if $\exists c \in \mu$ s.t. $c(S) = y$ and $c(S \setminus z) = x$ for some $S \in \Omega$.

Thm 3: *Let $\mathcal{B}_\mu$ be the betweenness relation associated with μ.*
(i) $\mathcal{B}_\mu$ satisfies B1 $-$ B3 iff $\exists$ ordering $>$ s.t. $\mu \subseteq \mu^\theta(>)$;
(ii) $>$ is unique (up to reversal) iff $\mathcal{B}_\mu$ satisfies sB1 & B3.

- Betweenness relations are used to axiomatize geometry.
- Huntington & Kline'1917 proposed 11 different sets of axioms to characterize the betweenness on a real line.

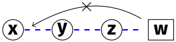
Thm 3: *Let $\mathcal{B}_\mu$ be the betweenness relation associated with μ.*
(i) $\mathcal{B}_\mu$ satisfies B1 $-$ B3 iff $\exists$ ordering $>$ s.t. $\mu \subseteq \mu^\theta(>)$;
(ii) $>$ is unique (up to reversal) iff $\mathcal{B}_\mu$ satisfies sB1 & B3.

B1. Each triple $x, y, z \in X$ appears in at most once in $\mathcal{B}_\mu$.

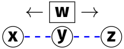
sB1. Each triple $x, y, z \in X$ appears once in $\mathcal{B}_\mu$.

For each distinct $x, y, z, w \in X$ s.t. $y\ \mathcal{B}_\mu\ \{x, z\}$,

B2. If $z\ \mathcal{B}_\mu\ \{x, w\}$, then it is not $w\ \mathcal{B}_\mu\ \{x, y\}$.



B3. If x, y, w and y, z, w are in $\mathcal{B}_\mu$, then
$y\ \mathcal{B}_\mu\ \{x, w\}$ or $y\ \mathcal{B}_\mu\ \{z, w\}$ but not both.



identification of $>$

Corollary: *$\mu = \mu^\theta(>)$ and $\mu = \mu^\theta(>')$ iff $>'$ is the inverse of $>$.*

In words: If a choice model μ coincides with the minimal extension of rational choice functions with respect to a primitive ordering $>$, then this primitive ordering is identifiable unique up to its inverse.

also in the paper

Robustness: Which choice models render unique orderly representations independent of the primitive ordering(s)?

also in the paper

Robustness: Which choice models render unique orderly representations independent of the primitive ordering(s)?

Defn: A choice model μ is universally self-progressive if μ is self-progressive wrt any domination relation $\rhd$ (that can be obtained from any set of primitive orderings $\{>_s\}_{s\in\Omega}$).

maximization of set contingent utilities

c : a complete contingent plan to be implemented

$U(x, S)$ be the set contingent utility of choosing x.

c : a complete contingent plan to be implemented

$U(x, S)$ be the set contingent utility of choosing x.

maximization of set contingent utilities

c : a complete contingent plan to be implemented

$U(x, S)$ be the set contingent utility of choosing x.

- each agent adopts a choice function by solving:

$$\max_{c \in \mathcal{C}} \sum_{S \in \Omega} U(c(S), S)$$

maximization of set contingent utilities

c : a complete contingent plan to be implemented

$U(x, S)$ be the set contingent utility of choosing x.

- each agent adopts a choice function by solving:

$$\max_{c \in \mathcal{C}} \sum_{S \in \Omega} U(c(S), S)$$

- the unique source of variation: multiplicity of maximizers.

maximization of set contingent utilities

c : a complete contingent plan to be implemented

$U(x, S)$ be the set contingent utility of choosing x.

- each agent adopts a choice function by solving:

$$\max_{c \in \mathcal{C}} \sum_{S \in \Omega} U(c(S), S)$$

- the unique source of variation: multiplicity of maximizers.

Convexity: if c^* is obtained as a “mixture” of some $c, c' \in \mu$, i.e. $c^*(S) \in \{c(S), c'(S)\}$ for every S, then $c^* \in \mu$ as well.

maximization of set contingent utilities

c : a complete contingent plan to be implemented

$U(x, S)$ be the set contingent utility of choosing x.

- each agent adopts a choice function by solving:

$$\max_{c \in \mathcal{C}} \sum_{S \in \Omega} U(c(S), S)$$

- the unique source of variation: multiplicity of maximizers.

Convexity: if c^* is obtained as a "mixture" of some $c, c' \in \mu$, i.e. $c^*(S) \in \{c(S), c'(S)\}$ for every S, then $c^* \in \mu$ as well.

- *meet* and *join* are special *mixtures*.

Prop: A choice model μ is universally self-progressive iff $\exists$ set contingent utility functions $\{U(\cdot, S)\}_{S \in \Omega}$ s.t. the maximizers of their sum comprises μ, i.e.

$$\mu = \mathrm{argmax}_{c \in \mathcal{C}} \sum_{S \in \Omega} U(c(S), S)$$

.

Prop: A choice model μ is universally self-progressive iff $\exists$ set contingent utility functions $\{U(\cdot, S)\}_{S \in \Omega}$ s.t. the maximizers of their sum comprises μ, i.e.

$$\mu = \operatorname{argmax}_{c \in \mathcal{C}} \sum_{S \in \Omega} U(c(S), S)$$

.

Prop: A choice model μ is universally self-progressive iff $\exists$ set contingent utility functions $\{U(\cdot, S)\}_{S\in\Omega}$ s.t. the maximizers of their sum comprises μ, i.e.

$$\mu = \mathrm{argmax}_{c\in\mathcal{C}} \sum_{S\in\Omega} U(c(S), S)$$

.

- to extend rational choice model into a universally self-progressive one, we must add every choice function.

Ex: choice with multiple rationales (Kalai et al.'02)

- Let $\{\succ_k\}_{k=1}^K$ be a set of strict preferences.
- Then, a choice function $c \in \mu$ if for each S, the alternative $c(S)$ is the $\succ_k$-maximal one in S for some k.
- To see that μ is universally self-progressive, define

$$U(x, S) = \begin{cases} 1 & \text{if } x = max(S, \succ_k) \text{ for some } k \in \{1, \ldots, K\}, \\ 0 & \text{otherwise.} \end{cases}$$

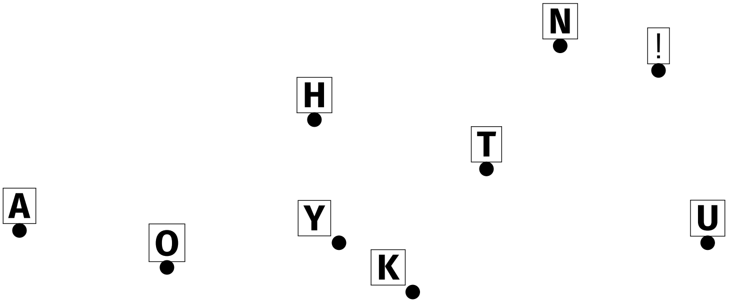
Ex: choice with multiple rationales (Kalai et al.'02)

- Let $\{\succ_k\}_{k=1}^{K}$ be a set of strict preferences.
- Then, a choice function $c \in \mu$ if for each S, the alternative $c(S)$ is the $\succ_k$-maximal one in S for some k.
- To see that μ is universally self-progressive, define

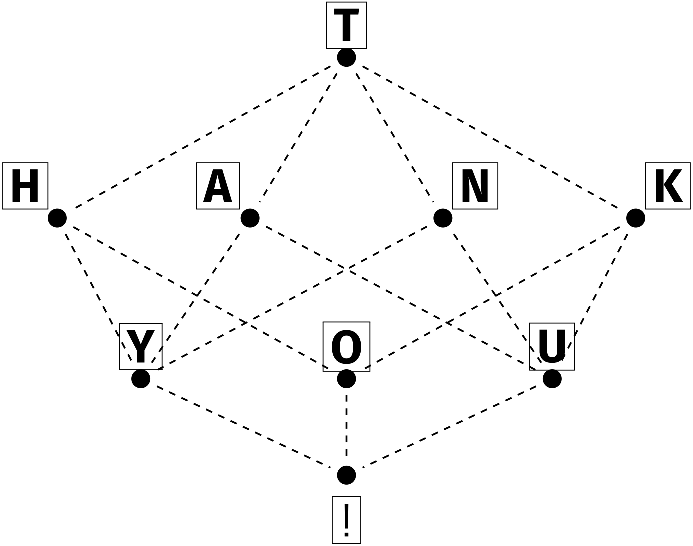
$$U(x,S) = \begin{cases} 1 & \text{if } x = max(S, \succ_k) \text{ for some } k \in \{1,\ldots,K\}, \\ 0 & \text{otherwise.} \end{cases}$$

conclusion

conclusion



conclusion



identification of $>$

We introduce conditions structuring the $\mathcal{B}_\mu$ that are necessary and sufficient for the existence and uniqueness of a primitive ordering $>$ that renders a choice overload representation to the choice model, i.e. $\mu \subseteq \mu^\theta(>)$

Corollary: *If a choice model μ coincides with the minimal extension of rational choice functions with respect to a primitive ordering $>$, then this primitive ordering is identifiable unique up to its inverse.*

$\mu = \mu^\theta(>)$ and $\mu = \mu^\theta(>')$ if and only if $>'$ is the inverse of $>$.

Ex 1: a model of *satisficing*

- consider a population with the primitive ordering $>$
- each agent i has the same preference relation $\succ^*$, but a possibly different threshold alternative x_S^i for each S.
- i chooses the $\succ^*$-maximal alternative in $\{x \in S : x \geq x_S^i\}$

Ex 1: a model of *satisficing*

- consider a population with the primitive ordering $>$
- each agent i has the same preference relation $\succ^*$, but a possibly different threshold alternative x_S^i for each S.
- i chooses the $\succ^*$-maximal alternative in $\{x \in S : x \geq x_S^i\}$
- Is this model self-progressive?

Ex 1: a model of *satisficing*

- consider a population with the primitive ordering $>$
- each agent i has the same preference relation $\succ^*$, but a possibly different threshold alternative x_S^i for each S.
- i chooses the $\succ^*$-maximal alternative in $\{x \in S : x \geq x_S^i\}$
- Is this model self-progressive?
  Yes, $\langle \mu, \rhd \rangle$ is a lattice:

$c^i \vee c^j(S)$ is the $\succ^*$-maximal alternative $\geq max(\{x_S^i, x_S^j\}, \geq)$

$c^i \wedge c^j(S)$ is the $\succ^*$-maximal alternative $\geq min(\{x_S^i, x_S^j\}, \geq)$