

Equilibrium in a civilized jungle

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joint with...



an overview

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the language $\mathcal{L} = \{\geq_\lambda\}_{\lambda \in \Lambda}$ is (for this talk) a set of strict orderings over the set of agents N

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- ▶ the language $\mathcal{L} = \{\geq_\lambda\}_{\lambda \in \Lambda}$ is the stock of criteria that can be used to justify choosing an agent from a group.
- ▶ for example, these criteria might rank the agents according to their economic status, intelligence, or level of education.
- ▶ the phenomenon that we are trying to capture is that the assignment of objects is not entirely based on who is stronger, but requires **some socially legitimate justification**.

example

Let $N = \{1, 2, 3\}$ and $X = \{a, b, c\}$. The preference profile (\succsim^i) , and the power relation \mathcal{P} are specified as follows:

\succsim^1	\succsim^2	\succsim^3	\mathcal{P}
a	b	a	3
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- ▶ in a **jungle equilibrium**, the assignment of objects is entirely based on who is stronger, so $x^3 = a, x^1 = b, x^2 = c$ is the jungle equilibrium.

civilized jungle = jungle + language

Let $N = \{1, 2, 3\}$ and $X = \{a, b, c\}$. The preference profile (\succsim^i) , the language $\mathcal{L} = \{\succeq_\alpha, \succeq_\beta\}$ and the power relation \mathcal{P} are specified as follows:

\succsim^1	\succsim^2	\succsim^3	\succeq_α	\succeq_β	\mathcal{P}
a	b	a	1	2	3
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- ▶ for a given assignment $\mathbf{x} = (x^i)_{i \in N}$, we denote the group consisting of agent i and the agents who envy him by $E(\mathbf{x}, i)$.

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C-equilibrium is an assignment \mathbf{x} such that each agent i :

- ▶ *is **justifiable** within the group $E(\mathbf{x}, i)$,*
- ▶ *is **stronger** than other agents who are **justifiable** within $E(\mathbf{x}, i)$.*

example

γ^1	γ^2	γ^3	\succeq_α	\succeq_β	\mathcal{P}
a	b	a	1	2	3
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- ▶ the jungle equilibrium $\mathbf{x} = [b, c, a]$ is not a C -equilibrium, in that:
 - since both 1 and 2 envy 3, we have $E(\mathbf{x}, i)$, but
 - 3 is **not justifiable** within $\{1, 2, 3\}$, although he is the **strongest**.
- ▶ indeed, there is no C -equilibrium here.

Question: Is there a connection between the power relation and the language that will be necessary and sufficient for the existence of a Pareto efficient civilized equilibrium?

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Definition: A power relation \mathcal{P} is \mathcal{L} -concave if it is consistent with the language in the following sense: for every $i, j \in N$, agent i is \mathcal{P} -stronger than agent j if for each criterion $\geq \in \mathcal{L}$, either i is more suited than j or i can point to an agent k who is weaker than himself(i) and more suited than j according to \geq .

\geq_α	\geq_β	\mathcal{P}
1	2	3
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- ▶ \mathcal{P} is not \mathcal{L} -concave since 3 is stronger than 1, although 1 can point to himself for α criterion and 2 for the β criterion.

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\geq_α	\geq_β	\mathcal{P}
1	2	1
3	3	3
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- ▶ \mathcal{P} is \mathcal{L} -concave since 1 is \geq_α -best agent and 2 is the \geq_α -worst agent.

example

γ^1	γ^2	γ^3	\succeq_α	\succeq_β	\mathcal{P}
<i>a</i>	<i>b</i>	<i>a</i>	1	2	3
<i>b</i>	<i>a</i>	<i>c</i>	3	3	1
<i>c</i>	<i>c</i>	<i>b</i>	2	1	2

existence: sufficiency

γ^1	γ^2	γ^3	\geq_α	\geq_β	\mathcal{P}
<i>a</i>	<i>b</i>	<i>a</i>	1	2	1
<i>b</i>	<i>a</i>	<i>c</i>	3	3	3
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Proposition 1: *If the power relation \mathcal{P} is \mathcal{L} -concave, then the Jungle equilibrium is the unique C -equilibrium.*

existence: necessity

- ▶ the \mathcal{L} -concavity of the power relation is essentially necessary for existence of a Pareto efficient civilized equilibrium.

Proposition 2: *Suppose that for every $i, j \in N$ who are ranked consecutively in \mathcal{P} , if $i D_{\mathcal{L}} j$ then $i \mathcal{P} j$. If the power relation not weakly \mathcal{L} -concave, then there is a preference profile (\succsim^i) such that there is no Pareto efficient C -equilibrium.*

└ a counterpart to the SWT ─

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- ▶ Thus, the authorities can induce any J-constrained efficient assignment by determining the power relation accordingly.

Proposition 3: *For every J-constrained efficient assignment \mathbf{x} , there is a power relation \mathcal{P} such that \mathbf{x} is a C-equilibrium in the civilized jungle with the power relation \mathcal{P} .*

concluding remarks

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thank you!



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