Choice with Affirmative Action

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joint with...

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Question: How to reconcile these two potentially conflicting objectives?

examples from applications

Priority orderings	*Minorities*
exam-score order in school choice	neighborhood students

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a merit order in job assignment	applicants with disabilities
time-of-application in visa assignment	highly educated applicants

school choice application

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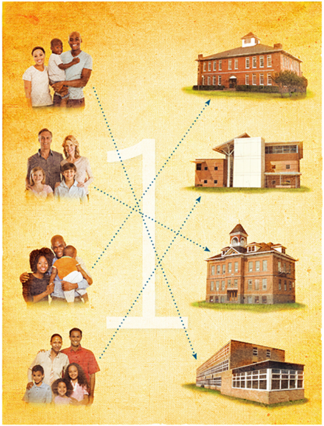

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- Although students' preferences are elicited, endowing each school with a choice rule is a part of the design process.
- When there are affirmative action concerns, which choice rule to use is non-trivial.
- Recent emprical studies, such as Leashno & Lo (2021), indicate that the design of the choice rule can have larger welfare implications than designing the rest of the mechanism.

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Outline

Part 1 The new framework and the ‘comparative statics’ axioms.

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Part 2 Relationship to the choice rules used in applications.

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Part 3 Some results from our ongoing research.

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A choice rule C for a school with q available seats (capacity) maps each problem $(S, \tau, \succ)$ to a nonempty subset $C(S, \tau, \succ) \subseteq S$ without exceeding the capacity i.e. $|C(S)| \leq q$.

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- **Priority-compatibility** A student is chosen over a higher priority student only if the former student is a minority student and the latter is a majority student.
- **Substitutability** A chosen student remains chosen when the set of students shrinks, everything else the same.

Substitutability is crucial because...

- Substitutable choice rules have been a standard tool following the seminal work of Kelso and Crawford, 1982, in broadening the classical matching model with single priority.
- Hatfield and Milgrom, 2005 show that substitutability guarantees the existence of stable matchings.
- Hatfield and Kojima, 2006 show that substitutability is almost necessary for the non-emptiness of the core in allocations problems
- Similarly, several classical results of matching literature have been generalized with substitutable choice rules (Roth and Sotomayor, 1990; Hatfield and Milgrom, 2005).

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priority improvements

- Given $(S, \tau, \succ)$, a priority ordering $\succ'$ is an improvement over $\succ$ for a student s, if when we move from $\succ$ to $\succ'$, the priority order of s strictly improves relative to at least one student, while the rest remains the same.

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$(S, \tau, \succ)$		$(S, \tau, \succ')$
1		3
2		1
3	$\longrightarrow$	2
4		4
5		5

Monotonicity in priority improvements

MPI: *If the priority order of a chosen minority student is improved, then chosen minority students remain chosen.*

$(S, \tau, \succ)$		$(S, \tau, \succ')$
1		3
2		1
3	$\longrightarrow$	2
4		4
5		5
$C(S, \tau, \succ) = \{1, 2, 3, 5\}$		$\{3, 5\} \subset C(S, \tau, \succ')$

type changes

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1		1
2		2
3	$\longrightarrow$	3
4		4
5		5
$C(S, \tau, \succ) = \{1, 2, 5\}$		$5 \in C(S, \tau', \succ)$

Underlying principle behind Monotonicity

Principle: Conceive affirmative action as a fixed limited resource where the *intended beneficiaries* are the minority applicants <u>who have relatively low priorities</u>.

Important: Whether a minority is eligible for affirmative action resources depends on the problem!

priority order of a chosen minority applicant is improved or the type of a chosen minority applicant is changed

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representation result

Theorem 1 *A choice rule is an affirmative action rule that satisfies the* **monotonicity axioms** *if and only if it admits a bounded reserve representation.*

reserve representation

A choice rule C admits a reserve representation via a reserve function R if for each problem $(S, \tau, \succ)$, $C(S, \tau, \succ)$ is obtainable as follows:

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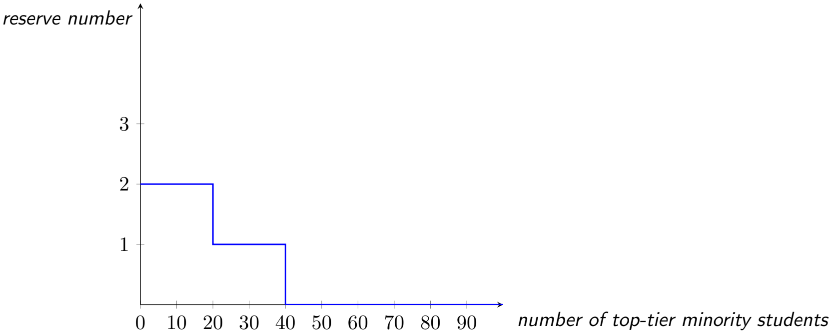
example: step-wise adjusted rule

Let $q = 100$. For each problem $(S, \tau, \succ)$, if the number of top-tier minority students is

- at most 20 (few), then the reserve number $R(S, \tau, \succ) = 2$;
- more than 20 but at most 40 (many), then $R(S, \tau, \succ) = 1$;
- more than 40 (enough), then $R(S, \tau, \succ) = 0$.



Part II: Choice rules in applications

Lexicographic affirmative action rules

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- school choice rules in Boston and in Chicago
- choice rules for Indian governmental job positions
- H-1B visa allocation for U.S. immigration
- Israeli “Mechinot” gap-year program

Although these applications include different institutional constraints, the lexicographic feature remains common.

lexicographic procedure

- Given a problem $(S, \tau, \succ)$, the affirmative (priority) ordering $\succ^a$ is obtained from $\succ$ by moving the minority students to the top of $\succ$,

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$\succ$		$\succ^a$
1		3
2		5
3	$\longrightarrow$	1
4		2
5		4

lexicographic procedure

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- In turn, the lexicographic procedure allocates seats sequentially according to the lexicographic order.

an example: no reserves

- For $l = [\text{open, open, open}]$

$(S, \tau, \succ)$	$\succ$	$\succ$	$\succ$
1	$\underline{1}$	1	1
2	2	$\underline{2}$	2
3	3	3	$\underline{3}$
4	4	4	4
5	5	5	5

$$\longrightarrow \quad C^l(S, \tau, \succ) = \{1, 2, 3\}$$

an example: reserve first

- For $l = [\text{reserve}, \text{open}, \text{open}]$

$(S, \tau, \succ)$	$\succ^a$	$\succ$	$\succ$
1	$\underline{1}$	1	1
2	5	$\underline{2}$	2
3	2	3	$\underline{3}$
4	3	4	4
5	4	5	5

$\longrightarrow \quad C^l(S, \tau, \succ) = \{1, 2, 3\}$

an example: reserve last

- For $l = [$open, open, reserve$]$

$(S, \tau, \succ)$	$\succeq$	$\succeq$	$\succeq^a$
1	$\underline{1}$	1	1
2	2	$\underline{2}$	$\underline{5}$
3	3	3	2
4	4	4	3
5	5	5	4

$$\longrightarrow \quad C^l(S, \tau, \succ) = \{1, 2, 5\}$$

lexicographic representations

A choice rule C admits a lexicographic representation if there exists a lexicographic order, such that for each problem $(S, \tau, \succ)$, $C(S, \tau, \succ)$ is obtainable via the associated lexicographic procedure.

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A characterization of lexicographic choice

so far...

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BRR=Basic Axioms + Monotonicity

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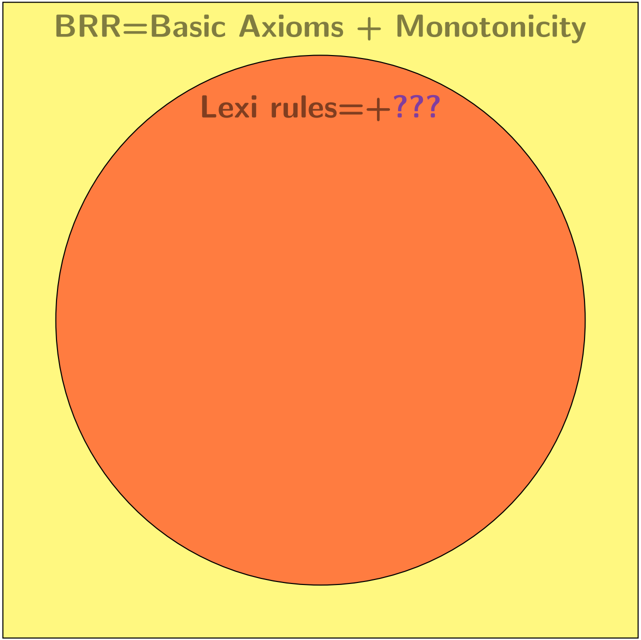

Consistency axiom

Consistency (in effective type changes): If changing the type of a chosen minority applicant is effective, i.e., if it results in new chosen minority applicants,

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Consistency (in effective type changes): If changing the type of a chosen minority applicant is effective, i.e., if it results in new chosen minority applicants, then changing the type of any chosen minority applicant with a lower priority must also be effective (unless all minority applicants are already chosen)

effective type changes

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$(S, \tau, \succ)$		$(S, \tau', \succ)$
1		1
2		2
3	$\longrightarrow$	3
4		4
5		5
6		6

$C(S, \tau, \succ) = \{1, 2, 3, 4\}$

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6		6

$C(S, \tau, \succ) = \{1, 2, 3, 4\}$ $\qquad$ $C(S, \tau', \succ) = \{1, 2, 4, 5\}$

Consistency

Consistency (in effective type changes): If changing the type of a chosen minority applicant is *effective*, then changing the type of any chosen minority applicant with a lower priority must also be *effective*.

$$
\begin{array}{c}
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\end{array}
\quad \longrightarrow \quad
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\end{array}
\quad \Longrightarrow \quad
\begin{array}{c}
\underline{(S,\tau'',\succ)} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6
\end{array}
$$

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$(S,\tau,\succ)$		$(S,\tau',\succ)$		$(S,\tau'',\succ)$
1		1		1
2		2		2
3	$\longrightarrow$	3	$\Longrightarrow$	3
4		4		4
5		5		5
6		6		6
$C(S,\tau,\succ)=\{1,2,3,4\}$		$C(S,\tau',\succ)=\{1,2,4,5\}$		$C(S,\tau'',\succ)=\{1,2,5,6\}$

Theorem 2: *A choice rule is an affirmative action rule that satisfies the* **monotonicity** *axioms and* **consistency** *if and only if it admits a lexicographic representation.*

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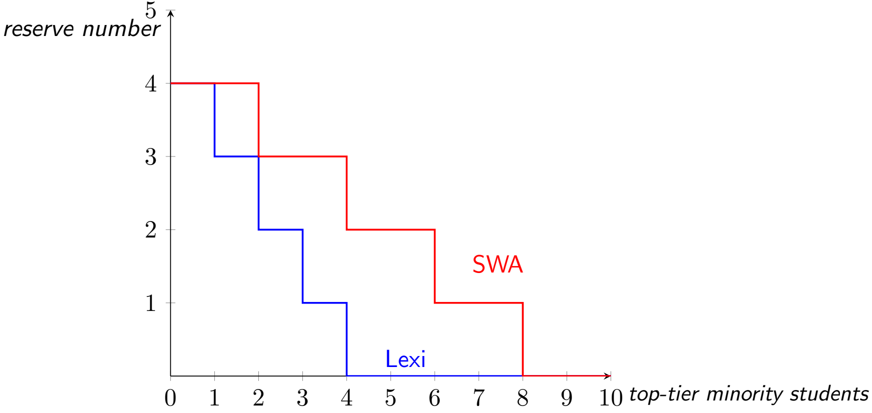
lexi vs. step-wise adjustment



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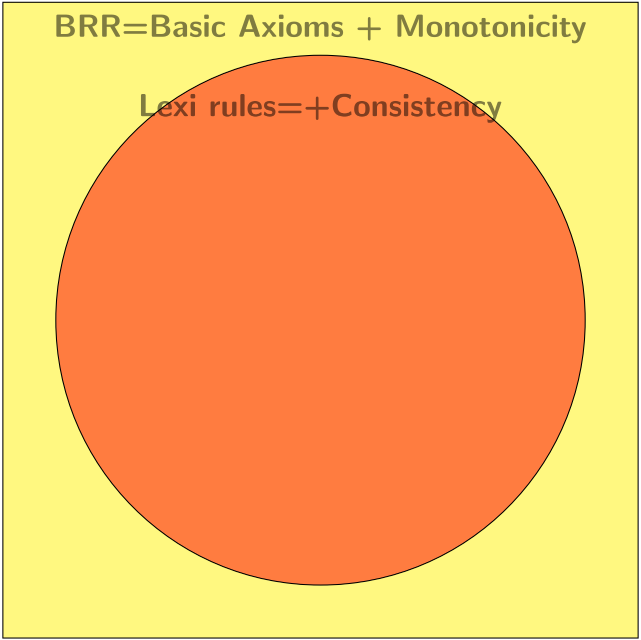

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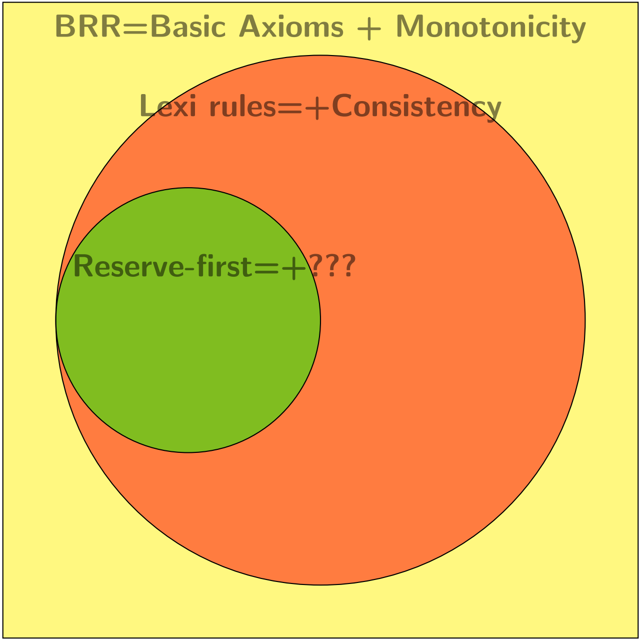

BRR=Basic Axioms + Monotonicity
Lexi rules=+Consistency
Reserve-first=+???

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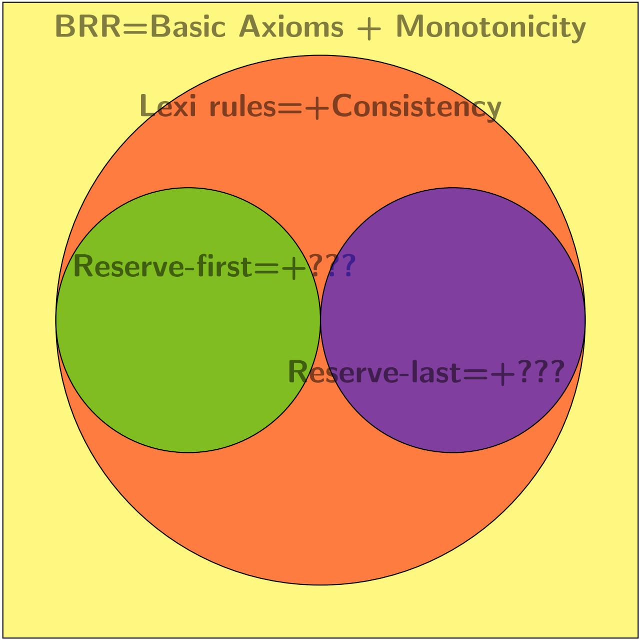

BRR=Basic Axioms + Monotonicity
Lexi rules=+Consistency
Reserve-first=+???
Reserve-last=+???

Part III: Some new results

a thought experiment

- If the whole purpose of affirmative action is to correct possible biases in the test scores, a natural affirmative action rule may decide to do:
  - a high level of affirmative action when minority students are relatively low ranked (which indicates a high level of concern for test-score bias), and
  - a low level of affirmative action (possibly none) when minority students are relatively high ranked (which indicates a low level of concern for test-score bias).

Revision (inverse monotonicity) axioms

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- Sheds light on how substitutability restricts affirmative action.

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conclusion: a menu for affirmative choice

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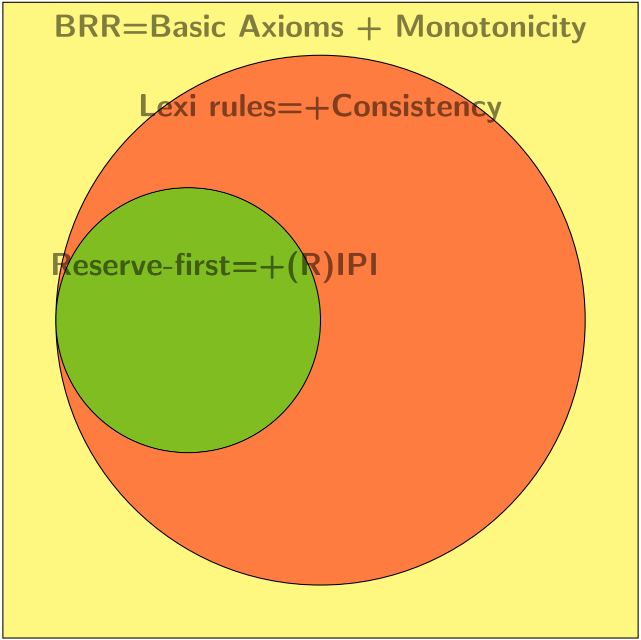
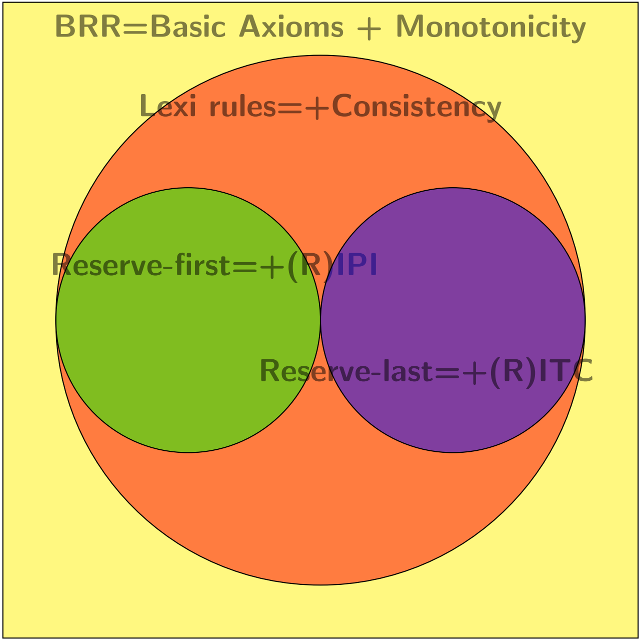


conclusion: a menu for affirmative choice



conclusion

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Monotonicity axioms

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Threshold-reserve representation

- **Conditional invariance under priority improvements (CIPI):** If changing the type of a chosen minority student is effective, then improving the priority of a lower ranked chosen minority student does not affect the choice unless he is moved above the former student.

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Theorem 3: *A choice rule C is an affirmative action rule that satisfies conditional invariance under priority improvements if and only if it admits a threshold-reserve representation.*